
0954.01002

Toth, Imre

Aristoteles and the axiomatic foundation of geometry. Prolegomena to the understanding of non-euclidean fragments in the "Corpus Aristotelicum" in their mathematical and philosophical context. (Aristotele e i fondamenti assiomatici della geometria. Prolegomeni alla comprensione dei frammenti non-euclidei nel "Corpus Aristotelicum" nel loro contesto matematico e filosofico). 2nd revised and corrected ed. (Italian)

Temi Metafisici e Problemi del Pensiero Antico. Studi e Testi. Milano: Vita e Pensiero. 701 p. (1998). [ISBN 88-343-0085-8/pbk]

From 1966 onward, Imre Toth argued in a number of publications that the mathematicians at Plato's Academy tried to prove, first directly and next indirectly, an equivalent to Euclid's fifth postulate (namely Elements I.29, cf. below). In this way, as he saw it, a counterpart of Saccheri's quasi-non-Euclidean geometry was created – a coherent deductive chain of propositions based on the remaining postulates and axioms and on the negation of the (equivalent of the) fifth postulate. He mainly argued from a set of passages in the Aristotelian corpus, which in his reading showed that a whole body of theorems belonging to such a chain was known.

This not fully unprecedented but still revolutionary thesis was mostly received with taciturn reservation, for which reason Toth is now publishing a book size discussion of the Aristotelian passages (which may have appeared, but which the reviewer has not seen); the volume under review is a kind of broad prolegomenon and philosophical commentary to the matter. The book falls in two parts, the first of which is meant as a survey of the origin of the search for axiomatics at the Academy (referring mostly to Platonic and Aristotelian texts and organized around the non-Euclidean problem); the second part is a protracted essay located in the boundary region between the philosophy of mathematics (centred on Euclidean, non-Euclidean and absolute geometry) and non-chronological history.

Unfortunately, the book is unconvincing when submitted to a close reading and checked against the sources. This follows in part from the essay style, where misquotations, reformulations and oblique allusions to the sources outweigh precise references, in part from what the reviewer cannot help seeing as distorted interpretations. Many of the philosophical reflections and observations are stimulating; however, the philosophical stance, in as far as it can be safely extricated from the poetical apparel, seems inconsistent. For reasons of space, a few illustrations of these objections will have to suffice (all quotations from the book and from Greek sources are translated into English by the reviewer).

1. On p. 585 (and already, slightly less sharply, on p. 564), it is stated that Aristotle gives absolute priority to the object which is known over the knowledge about the object. The actual claim of the passage referred to (Categories 7^b23-27) is that the object will generally have ontological priority, and that our knowledge will come into being together with its object "in few or no cases".

2. Elements I.29 is referred to repeatedly, often obliquely; however, the first time its

content is explained (p. 100) it misquoted as "if two straight lines are parallel, then they are co-orthogonal". Actually, the proposition deals with the angles that are produced if a pair of parallels is cut by a third line; co-orthogonality follows without difficulty, but is not mentioned at all by Euclid. As a result, the discussion of a purported fallacy in the proof (rejection of the elliptic geometry but not of the hyperbolic possibility) on pp. 465ff is wholly off the point – all Euclid does when two symmetrically located and thus equivalent angles are supposed to be unequal is to assume that a specified one of the two is taken to be larger.

3. The supposedly most striking proof that Aristotle knew about (quasi-)non-Euclidean geometry is a passage from the Eudemian Ethics (1222^b15-37, repeated in the post-Aristotelian epitome Magna moralia $1187^a36-{}^b3$). It is asserted (in these words on p. 584, but equivalently elsewhere) that "in order to illustrate the concept of preferential choice, Aristotle does not cite an example drawn from the domain of ethical or political praxis. Unexpectedly, even surprisingly: the only example comes from the domain of geometry. And it is the alternative between a Euclidean and a non-Euclidean triangle". What actually goes on is very different: Aristotle wants to illustrate in a simple way (referring for deeper explanation to the Analytics) the relation between basic principles (archai) and their consequences. The example is that the sum of the angles in a quadrangle (4 right angles) is a consequence of the sum of the angles of the triangle; if the latter were to change (metaballō – thus not "if it were different"), for instance into three right angles, then even the former would change (viz into 6 right angles). This geometric observation is in need of no axiomatic network; it follows from the drawing of a diagonal in the triangle. Aristotle does not explain this, but obviously expects the derivation to be something simple which his audience (not familiar with the technicalities of the Analytics) understands. Moreover, is it explained that if the sum π of the angles of the triangle did not follow from other reasons (which it is thus supposed to do), then this would have the role of a first principle.

4. On p. 526 it is stated that Metaphysics 1052^a4-7 asserts that "Euclidicity and non-Euclidicity are invariant properties, immutable, of each its own universe, since it cannot happen that one triangle be Euclidean and another non-Euclidean or – to speak in terms of time, which anyhow brings the same result – that a triangle may sometimes be Euclidean and sometimes non-Euclidean". What Aristotle actually says is simply that "if we assume that the triangle does not change, then we shall not assume that at some times it possesses two right angles and at same times not (for this would mean that it changed)".

Many other examples are of the same kind. Several chapters are spun over the assumption that Plato's Cratylus deals with the question whether the internal coherence of non-Euclidean geometry suffices for making it true (supposed to be what Cratylus really means when claiming that the sounds or letters of words determine their meaning), or mathematical truth has to be guaranteed in a different way (assumed to be what Socrates means to demonstrate when destroying Cratylus's phono-semantics by counterexamples).

One passage remains which to a first reading might seem to lend some support to the thesis, namely On the Heavens 281^b2-7, which is taken on p. 109 to "assert the existence of squares with commensurate diagonal" (similarly passim), and on p. 539 to present the impossibility "of a triangle to have the sum of the angles equal to 2R" as an example of a merely "hypothetical impossibility". But even though this agrees with current translations, the non-Euclidean implications are dubious. The passage distinguishes things that are false haploos, "taken in isolation", from those which are false "from a foundation" (ex hypotheseōs); the most plausible reading of the passage is thus simply that the incommensurability of the diagonal and the sum of the angles of the triangle are not independent or primary facts but consequences of prior principles (cf. what was said above on Eudemian Ethics $1222^b 15-37$).

It remains that Prior Analytics $65^a 4-9$ criticizes "those who suppose they draw parallels" using the sum of the angles of the triangle, which sum on its part is only established on the assumption that parallels can be drawn; and that Plato (Republic 533C) values mathematics less than dialectic in the education of the guardians because the reasoner in mathematics does not understand the starting point or archē while dialectic is supposed to get beyond this kind of unproved first principle or "hypothesis"; but control of Toth's impressive body of textual hints and references left the reviewer unconvinced that this indubitable awareness of the conditions of axiomatic thinking (which Toth is not the first to recognize) led to the creation of any kind of quasi-non-Euclidean geometry.

As to the apparent philosophical inconsistencies (which may however be mere consequences of polemically intended eclecticism), one example shall suffice. Mostly, "Euclidean and non-Euclidean knowledge" are supposed to possess no truth value within the absolute geometry encompassing both, to "describe no preexisting object", and to constitute only "linguistic objects" once articulated (in these words p. 564); but a passage blaming Aristotle for not understanding that auxiliary lines that can be added to a diagram in a proof are present in the diagram actually, not only potentially (p. 520) asserts that all auxiliary lines and (in less clearcut words) all geometric figures are timelessly present in "actual being"; since the auxiliary line in question cannot be constructed in an elliptic geometry, this claim of absolute existence must concern the three geometries separately. (Most likely, a precise formulation of the vague statement would entail paradoxes similar to those familiar from set theory).

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Keywords : Aristotle; Plato; Euclid; non-Euclidean geometry
Classification :
∗01A20 Greek or Roman mathematics
51-03 Historical (geometry)

Cited in ...