Early Mesopotamia:

A statal society shaped by and shaping its mathematics

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Gino Arrighi in memoriam

I. State and mathematics

Since what feels like times immemorial, "the state" as well as "mathematics" exist. As a rule, the relations between the two at a given moment appear to be accidental; only slowly are their "essences" shaped by the interaction.

At the emergence of the state as a type of social organization, the situation was different. Most statal systems have originated in complex processes, either as "pristine states" via expanding chiefdoms or as "secondary states", in interaction with (often as military protection against) existing states. As a rule, the involvement of anything than can be considered as mathematics in such processes has been peripheral, if not totally absent.

In a few exceptional cases, however, mathematical technologies have played a major role in the shaping of the state (and have, in consequence, themselves become more sophisticated in the process, developing into recognizable *mathematics*).

One instance of such an intimate bond is that between the Inca state and its accounting. I know too little about the matter to go into details – I suspect, moreover, that the general knowledge of the topic is insufficient to trace the connections between the development of the state and of the *quipu* system.

Possibly, another instance is constituted by the relation (which, however, may be less pivotal) between the Maya states and their "chrono-theology"; even here I abstain from further discussion for lack of deeper knowledge. In any case, the Maya state formation is not pristine.

A third instance, perhaps the most indisputable case and at least the one which is best reflected in the sources (though still indirectly), is offered by the formation of states in southern Mesopotamia.

Prolegomena

Before approaching the subject-matter itself, something must be said about the notion of "mathematics", and about the notion of a "state".

For the present purpose, the transition to "recognizable mathematics" may be characterized as

> the point where *pre-existent and previously independent* mathematical practices are coordinated through a minimum of at least intuitively grasped understanding of formal relations.

Political anthropologists have discussed the emergence of *statal organization* of society in different terms, not necessarily as mutually exclusive as often assumed in the debate. According to Morton Fried's classic *The Evolution of Political Society* [1967: 235], the state arises as

> a collection of specialized institutions and agencies, some formal and others informal, that maintain an order of stratification

"stratified society" being a society [*ibid.*, p. 186]

> in which members of the same sex and equivalent age status do not have equal access to the basic resources that sustain life

This stratification may come about in several steps ("big man", chiefdom spurred by warfare, leading to a three-class division into slave owners, commoners, slaves).

Elman Service's emphasis in *Origins of the State and Civilization* is different, seeing [1975: 305] statal organization as resulting through a quantitative and often gradual development from

> relatively simple hierarchical-bureaucratic chiefdoms, under some unusual conditions, into much larger, more complex bureaucratic empires

The chiefdom itself is understood as a hierarchical organization legitimized by social *functions* wielded by the chief for common benefit (according to Service mostly functions of a redistributive nature) in a theocratic frame of reference, where

> economic and political functions were all overlaid or subsumed by the priestly aspects of the organization

A number of other, less abstract discussions of the early state are regionally focused. One of these is Runciman's "Origins of States: The Case of Archaic Greece" [1982]. According to his view, "the emergence of a state from nonstate or stateless forms of social organization" should be characterized by

> these necessary and jointly sufficient criteria:
>
> Specialization of governmental roles; centralization of enforceable authority; permanence, or at least more than ephemeral stability, of structure; and emancipation from real or fictive kinship as the basis of relations between the occupants of governmental roles and those whom they govern.

In an article on “Population, Exchange, and Early State Formation in Southwestern Iran”, in which Henry T. Wright and Gregory A. Johnson try to base themselves “on the total organization of decision-making activities rather than on any list of criteria”, the state is described [1975: 267]

> as a society with specialized administrative activities. By ‘administrative’ we mean ‘control’, thus including what is commonly termed ‘politics’ under administration. In states as defined for purposes of this study, decision-making activities are differentiated or specialized in two ways. First, there is a hierarchy of control in which the highest level involves making decisions about other, lower-order decisions rather than about any particular condition or movement of material goods or people. Any society with three or more levels of decision-making hierarchy must necessarily involve such specialization because the lowest or first-order decision-making will be directly involved in productive and transfer activities and second-order decision-making will be coordinating these and correcting their material errors. However, third-order decision-making will be concerned with coordinating and correcting these corrections. Second, the effectiveness of such a hierarchy of control is facilitated by the complementary specialization of information processing activities into observing, summarizing, message-carrying, data-storing, and actual decision-making. This both enables the efficient handling of masses of information and decisions moving through a control hierarchy with three or more levels, and undercuts the independence of subordinates.

Though meant to be generally useful, the description *is* specifically geared to what happened when statal systems emerged in southern Mesopotamia and southwestern Iran.

The West-Asian “token system”

Central to that “control” which Wright and Johnson speak about is the “token system”, an accounting system based on small and less small cones, spheres, discs, tetrahedra, rods etc. made of burnt clay – often (and to some extent since the documented beginnings) with markings that constitute sub-types. The system turns up in Syria and Western Iran around 8000 BCE, concomitantly with the agricultural/ revolution, spreading during the following millennia to the whole region reaching from south-eastern Anatolia and Palestine to the Indus area, and remaining alive until the early third millennium BCE. Though some suggestions pointing in the same direction had been in discussions of late fourth-millennium Iranian material, the discovery of the system and of its chronological and geographical reach is unambiguously the merit of Denise Schmandt-Besserat. Her first publication on the topic is from [1977]; a complete survey of her results and interpretations is the double volume *Before Writing* [1992].

According to its use in the fourth millennium and to continuity with proto-cuneiform writing, the various tokens served to represent quantities (probably standard containers) of grain, oil, etc., and heads of livestock – perhaps also quantities of work.

For a number of reasons, the original social function of the system can hardly have been inter-community trade (which did exist, as documented by the spread of obsidian). First of all, any use of quasi-monetary symbols without tangible value (paper money, bills of exchange) presupposes banks and police forces which can enforce the obligations they represent. Moreover, the tokens were simply thrown out once they had been used, which excludes even a local monetary function.

Instead, the use of prestige versions (made from marble, alabaster, etc.) as grave-goods in high-prestige graves [Schmandt-Besserat 1988] and the presence of tokens in communal storehouse areas suggest that the tokens functioned as means of accounting in a redistribution system, and that management of this redistribution system carried the highest social prestige – cf. Elman Service as quoted above.

In this connection, two observations should be made:

– Redistribution within the community is very common in pre-state societies, but redistribution built on detailed accounting is rather unique. If Inuit hunters kill a walrus and give others access to the meat, this is done from an expectation of reciprocity, and on the part of the more skilled hunters in expectation of prestige; but in neither respect is detailed accounting involved, nor can it be.
– This type of accounting can doubtlessly be characterized as a *mathematical technique*. But we have no evidence for numerically standardized bundling of units (but actually some counter-evidence from the fourth millennium, cf. below). It is therefore most likely that (e.g.) a small token corresponded to a specific customary basket containing grain, (e.g.) a small sphere to some larger equally customary container, and that the ratio between the two was not numerically but physically (that is, not precisely) fixed. In other words, the mensuration inherent in the token system appears not to have been coordinated neither with the actual bundling levels of an oral counting system nor with the very principle of numerical bundling; if this is so, the system is hardly an instance of *mathematics* in the above sense.

Fourth-millennium developments

In the earlier fourth millennium, the city Susa in a river valley in the Zagros area in southwestern Iran became the centre of a wider settlement system, in which the redistribution system developed into what looks most of all as a payment of tribute or taxes to the central temples of Susa. In this context, the tokens were put to new use: enclosed in hollow clay envelopes ("bullae"), they appear to have served as bills of lading for goods delivered from the periphery to the centre. This goes hand in hand with the development and refinement of other bureaucratic devices and procedures – not least the use of cylinder seals as "certifiable signatures" of particular officials or offices. Since the contents of bullae could only be "read" if they were broken (and thus no longer controllable), impressions (or representative pictures) of the tokens to be put into them were made on the surfaces before they were closed and sealed.

A similar social development may have started slightly later in Uruk in the Mesopotamian South, but it soon went much further. The reason was that a climatic change and lowering of the water level in the Gulf opened the possibility for irrigation agriculture in the future Sumerian area, allowing a violent expansion of both agricultural output and population – see, e.g., [Nissen 1988: 58–61].

Probably in an initial phase, it was discovered that impression or depiction of the tokens on the surface of bullae made it possible to dispense with the contents, and to use a flattened piece of clay as carrier of the impressions / depictions.[1] Very soon (c. 3300 BCE[2]), genuine writing was also invented – and truly *invented*, in one leap or at least in a very speedy process (no "primitive" precursor steps are known).

The "proto-literate" script was ideographic, and used composition in a way that is very close to what is found in pidgin and creole languages.[3] Most signs (traced by means of a pointed stylus) were directly pictographic, showing for instance a jar, a head, the mountains to the east, the sun rising between these, etc. Some, however, depict *tokens representing the thing* instead of the thing itself. Quite striking, and enigmatic until the discovery of the token system, is the sign for a sheep: a circle marked by a cross. As a matter of fact it does not depict the animal but the token standing for the animal.

In contrast to these drawings of things or tokens, metrological and numerical units were *impressed* by a different stylus, as representations of tokens. This stylus was cylindrical, thick in one end and thin in the other. Impressed vertically it might produce a large or a small circle, oblique impression could render a large or a small cone.

The proto-literate script did not attempt to render the sentences of spoken language. Some 85% of the surviving texts are accounts made in fixed formats, rather to be likened to a statistical table or a ledger than to literary texts. The remaining 15% are "lexical lists" which served to teach the script. Writing was invented in Uruk, but the idea and the bureaucratic use (not the script itself) were soon borrowed into Susa and a number of other Iranian localities which formed a shared cultural system.

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[1] These "numerical tablets" provide the evidence referred to above that no fixed bundling system was yet in existence around the mid-fourth millennium BCE.

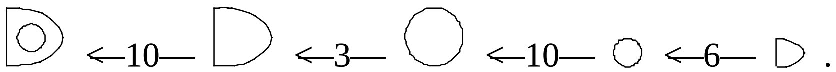
[2] From this point onward, I follow the "middle chronology", as used, e.g., in [Liverani 1988].

[3] This statement should not be taken as a claim that the inventors of the "proto-literate" script spoke a pidgin – the patterns in sacred architecture shows cultural continuity over about 2000 years preceding the invention, and thus continuity of the culturally hegemonic stratum of the area. But the principles of the invention may have been inspired by familiarity with a pidgin spoken by enslaved populations – cf. imminently.

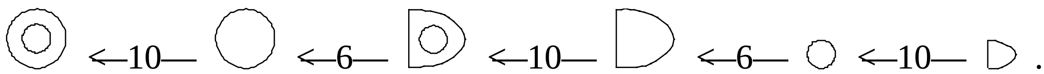
The proto-literate Uruk metrologies

In the numerical and metrological sequences of the Uruk writing system, bundling was numerically fixed.[4]

One sequence was used for the measurement of grain, and may reasonably be considered a derivation from the traditional use of tokens. The “basic unit” in this system, a depiction of a small cone, was . 6 of these became , a picture of a small sphere. 10 small spheres were , a picture of a large sphere. 3 large spheres were bundled as , a picture of a large cone. 10 large cones, finally, became , possibly a picture of a punched large cone (an existing token), but perhaps a new construction made in parallel with the number sequence. In a notation due to Jöran Friberg, the sequence as a whole looks as follows



Another sequence was used for counting most types of discrete items, and may be regarded as a “number sequence”. Whereas the grain sequence is likely to express an old system in a new medium, the number sequence will be new – the representation of pure numbers (i.e., numbers abstracted from the quantity they count) by tokens will have had no purpose, at least not before their inclusion in bullae. The corresponding diagram is



This sequence, in contrast to the preceding one, is highly systematic, and therefore almost certainly represents a deliberate transformation of the grain sequence made so as to fit an existing oral number system, and perhaps extending it beyond existing spoken numerals. As we see, the signs for 600 ( ) and 36000 ( ) are produced by superposition of 10 ( ) on 60 ( ) and 3600 ( ), respectively, while 60 ( ) is chosen as an “enlarged” unit ( ).

The latter feature suggests that the spoken numeral system treated the step 1—>10 differently from the step 10—>60 (if not, there would be no reason to invert the order of and in the grain system). Evidently, the “second return” of the unit as 3600 could not repeat the system, the “number-and-measure” stylus having only two ends, each of which could be impressed vertically or obliquely. In consequence, the

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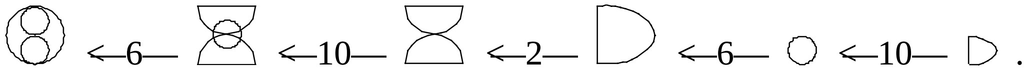
[4] For the following description of the metrological and numerical sequences I build on [Damerow & ENglund 1987].

written system gives no clues as to whether 3600 was already a unit in the spoken system.

For specific counting purposes – apparently the counting of bread or grain rations, perhaps also portions of dairy products – a particular “bi-sexagesimal system” with the following structure was in use:

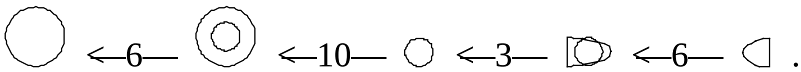


The agreement with the lower orders of the “general” counting system suggests the bisexagesimal system to have been shaped so as to fit particularly bureaucratic procedures or habits. Such an adaptation recalls our counting sheets of paper in units of 500, bottles of wine in dozens, etc., sometimes but not always corresponding to standard packages – such adaptations are amply present in the later Mesopotamian record.

We might be tempted to conclude from the split between the two counting systems after the level of 60 that the level 3600 did not exist in the spoken number system but was a product of the new bureaucratic device; the existence of the medieval “hundredweight”, deviating from the pre-existing 100 for similar reasons, shows that this conclusion is not warranted.

Two other metrological sequences exemplify the converse process, the adaptation of administrative procedures to mathematical structures. One is the area system, the other the administrative calendar.

The structure of the area system in itself shows little mathematical system:



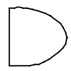
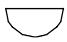
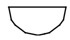
Such lack of matheamtical system is in itself an indication that the system is a normalization of a pre-existing system of “natural” (irrigation, seeding or similar) measures – a conclusion which is supported by linguistic arguments [Powell 1972, *passim*]. There is no direct proof of it, but it is a fair assumption that the system (which coincides with what is still known and well documented in much later periods) was already geared to the length metrology (in which lengths are given in “pure” numbers referring to the unit nindan or “rod”) – not least since it is almost certain that the area of slightly irregular rectangular fields was already determined as average length times average width, which would make no point if area units were not derived from lengths units. (the iku of later times) would then be the square on 10 nindan, a rectangle contained by 10 and 60 nindan.[5] On this foundation we may conclude

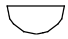
[5] The definition of area units in terms of linear metrology presupposes a conceptualization of area linked to square and rectangular shapes with measured sides; that this conceptualization was at hand, however, is not subject to doubt. Firstly, a number of prestige buildings in the area from this and earlier periods exhibit clearly rectangular layouts – a number of specimens are rendered in [Aurenche 1981]; secondly, the dimensions at least of certain buildings from

that the area metrology presents us with a deliberate coordination of several mathematical techniques and with implementation of the result in the administrative procedures concerned with the allotment of land in arithmetically determined proportion (which, without this new tool, could not be made).

The administrative calendar – functioning alongside the true luni-solar calendar with its months of changing length and its insertion of intercalary months when such turned out to be needed, which remained in use for ritual and time-keeping purposes – counted each month as if it consisted of 30 days, and each year as 12 months. It served for the calculation of fodder to be allocated to herds and, at least in later times, of the work which overseers were to press out of their crew each month irrespective of its length. Even in this case, only the introduction of a mathematical tool made possible the system of intense administrative control of subordinate staff

Still other metrological sequences were in use – most of them derive from those already mentioned by means of various kinds of extra marks (similar to those that had served in the token system), and serving for instance to count malted instead of ordinary grain. There is no reason to describe them in detail.

One common feature of all sequences, worth mentioning but so far not mentioned, was the way they were provided with subunits below . In all cases, the first level of sub-units was obtained by rotating either this sign or a shortened  90° clockwise,  and , respectively –  standing apparently for a halving (except when a day is seen as a sub-unit of an administrative month),  for a division into 5 parts.

It is possible that *one* of these subdivisions precedes writing –  could well be a depiction of a hemisphere, one of the old tokens. But , a mere rotation of , can hardly correspond to a particular token, nor can a rotation correspond to any feature of the token system. Globally, the way sub-units are formed thus reflects an underlying general idea of "forming sub-units".

Another general feature to be observed has to do with the function of the counting sequence. Freely movable tokens had to represent both the *kind* of thing they stood for and the quantity involved. In writing, it became possible to separate the two, combining, e.g., the ideogram for a sheep with the number «2» – which was indeed done. The mental habit involved in this splitting of quality and quantity also underlies the way the "lexical lists" were constructed from which the script was learned: in Luria's terminology, it reflects "categorical classification" and not "situational thinking". A plough will thus appear in a list of wooden objects, not together with

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the proto-literate period are determined in terms of an identifiable length unit – see [Beale & Carter 1983].

the ploughman or the grain. In one list – the "profession list" – the Cartesian product is not only an external condition but also involved into the structure of the list itself, which confronts field of activity with the hierarchy of positions. Even the orderly formats of bureaucratic accounting reflects the same mental habit.

The splitting into a Cartesian product of quantity and quality was not followed rigidly: if context determined quality unambiguously, this was not indicated – if a number stood for the length or width of a field, the unit nindan was thus omitted. This should not be understood as an indication of "primitivity" but as an instance of economical flexibility of thought: exactly the same thing happened when Stevin's decimal fractions came in common use, and his 375⓪7①2② was reduced to 375.72.[6]

Even this principle of economy can be seen in the light of Luria's dichotomy: Situational thinking is the habit of those whose world is largely made up of fixed situations, categorical classification is needed by those whose existence is less predictable – but in situations that *are* predictable, there is no reason they / we should not to resort to the simpler pattern. True mental flexibility encompasses the possibility to switch when convenient to subordinate patterns which, if they were hegemonic, would *not* be flexible.

Uruk: A "mathematical state"

If the emergence of mathematics proper is understood as the coordination of "*pre-existent and previously independent* mathematical practices [...] through a minimum of at least intuitively grasped understanding of formal relations", then there is no doubt that mathematics had started its career, if not before, then at least in late preliterate or proto-literate Uruk (and Susa) – nor that it were the needs of the administration of the new social system that asked for the creation or further unfolding of mathematics.

More interesting is perhaps the converse observation. The use of the mathematical tool was no instance of pure "technical rationality", the creation and implementation of means for an already established end which itself is not touched. If we compare the Uruk and subsequent Mesopotamian state formations with other early states, the end itself (the Mesopotamian state) can be seen to have been shaped by the means, no less than the successful appeal to military means may lead to the transformation of the state that appealed to it.[7]

A rash statement of this kind must evidently be explained. Redistributive systems are found in many pre-state societies; they correspond to the need for mutual support,

[6] This theme is further explored in [Høyrup 2000b].

[7] Keeping away from real politics we may think of Kästner's nightmarish poem about what would have happened to Germany after WW1 "Wenn wir den Krieg gewonnen hätten".

and may thus be said to correspond to a notion of social justice. However, this notion of justice cannot easily be carried over to the proto-statal situation. In Robert Carneiro's words [1981: 58], "what a chief gets from redistribution proper is esteem, not power"; further on (p. 61) Carneiro observes that

> As long as a chief merely returns everything he has been handed, he gains nothing in wealth or power. Only when he begins to keep a large part of it, sharing with his retainers and supporters but not beyond that, does his power begin to augment.
>
> But the power of a chief to appropriate and retain food does not flow automatically from his right to collect and redistribute it. Villagers freely allow a chief to equalize each family's share of meat or fish or crops through redistribution because they benefit from it. But they will not willingly suffer the same chief to keep the lion's share of food for himself. Before doing this, he must acquire additional power, and that power must come from some other source.

Since power only results when redistribution proper (where the chief retains only a small percentage of what passes through his hands) is transformed into *tribute* or *taxation*, where he keeps a large part for himself and for the "core of officials, warriors, henchmen, retainers, and the like who will be personally loyal to him and through whom he can issue orders and have them obeyed" [*ibid.*, p. 61], neither the commoners nor the chief and his circle have any reason the conceptualize the situation in terms of justice.

In the Susa-Uruk area, the situation will have looked differently, even though realities may have been similar. As shown by the use of *bullae* and by the accounting tablets, taxation and allocation of resources – be it the fields of high-ranking temple officials, be it the rations of grain given to workers – were made according to mathematically fixed rules. In this way, statal power was structured around and, apparently, legitimized by a transformed concept of "justice". Since accounts and lexical lists constitute our only written sources, we have no direct evidence for how the situation was conceptualized at the time; but literary evidence from a time when lexical lists from the proto-literate period were still in use indicates that at least the higher literate stratum thought of statal power in such terms.

A striking contrast is offered by the "nearest neighbour": Egypt. All evidence suggests that the Pharaonic state was legitimized by conquest, and (at least in the view of the literate) by a religious guarantee of cosmic order. Already during the First Dynasty, it is true, the yearly level of the Nile was recorded, in all probability in order to allow calculation of the taxation level of the year to come, and a biennial "counting of the riches of the land" was introduced.[8] But a biennial counting certainly does not allow any specific determination of dues and rights, nor is there any evidence that the measured Nile height served such purposes. "Justice" has no place in the

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[8] Nile observations as well as countings are documented on the Palermo Stone – translated, e.g., in [Clagett 1989: I, 67–95].

picture of early Pharaonic Egypt.

"Real justice"

"Real socialism" did not coincide with what had been proclaimed in programmes, and the real feudalism of the Middle Ages was conspicuously different both from Charlemagne's blueprint and from Fulbert of Chartres' theory of the respective roles of the praying, the warring and the labouring order. Likewise, mathematical "justice" (however much unequal) was certainly not the whole truth about the Uruk state. But it remains an essential part of the truth, and it conditioned the Mesopotamian states at least until the mid-second millennium BCE.

That it was only *part* of the truth, belonging rather on the level of hegemonic ideology than on that of social realities, can be seen from the preferred motif of the seals of high officials (found on no less than half of all early Uruk seals; two specimens are reproduced in [Nissen, Damerow & Englund 1993: 16]): A high official or priest looking on while overseers beat up pinioned prisoners. It is not unlikely that the violent increase in population did not result for local breeding alone but also from enslavement of significant populations from the mountain areas to the east – the pictograms for male and female slaves are indeed composed of an indication of sex (of a person) with a picture of the mountains[9] – and that this was brought about by the same climatic change as had made possible the irrigation revolution in the lowlands.

Such a hypothesis is supported by linguistics: many features of Sumerian look like those of languages that have developed from pidgins and creoles. If this is so, a large part of the population in the area will at one moment have spoken different languages, as did those of the West Indian plantations in the era of plantation slavery. In the longer run, even the masters will have taken over the creole created by the slaves.[10]

The Early Dynastic and Sargonic periods

The proto-literate period may have lasted from c. 3300 BCE to c. 2900 BCE (falling in two distinct sub-periods, "Uruk IV", 3300–3100, and "Uruk III" or "Jemdet Nasr", 3100–2900). It was followed in the Sumerian area (now doubtlessly Sumerian) by the "Early Dynastic Phase" (subdivided into ED I, ED II and ED III), c. 2900–2750–2600–2350 BCE.

In this phase, what seems to have been a social system with one major centre (Uruk) was replaced by one consisting of competing city states; and what looks like

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[9] No. 50 and 558, respectively, in [Labat 1963].

[10] This theme is explored in depth in [Høyrup 1992].

a state centred around a staff of high temple officials developed into states governed by a king.

From ED I we have no written sources, and from ED II very few. In ED III, their number proliferates; the continued use of the old lexical lists demonstrates continuity not only of the writing system but also of the school tradition. In the 26th century, however, a new phenomenon can be observed. Writing was now in wider use, serving also, e.g., for the stipulation of private contracts; concomitantly, and in consequence, the circle of the literate became broader. For the first time the group of scribes (dub.sar) turns up as a distinct *profession* (a single occurrence of the term in a Jemdet-Nasr texts shows that the profession was not hidden in earlier times under a different, un-deciphered name.

At the same time, and in all probability as a consequence of this, the script was put to new uses. We find the first literary texts – a proverb collection, a hymn – and the first instances of "supra-utilitarian" mathematical school problems. In contrast, all mathematical texts from the proto-literate period that can be identified as school texts are "model documents", distinguishable from real administrative texts only by the absence of an office seal and by the occurrence of numbers that are suspiciously round or nice and at times suspiciously large.

Rising city walls show clearly that warfare was an endemic condition of the ED-period, and that the king was a military leader. The many killed servants that followed their master to the underworld in the Royal Cemetery of Ur also demonstrate that the king had left behind any idea that he was the servant of society – no doubt that he was its overlord, and society a tool for his greatness. Only the very end of the ED period gives us written evidence, if not of the ritual slaughter of servants then at least of military activities; until then, even royal inscriptions show the king solely as the benefactor of temples and agricultural welfare (in strong contrast to early documents from Pharaonic Egypt). Literacy, so it appears, only reflects the functional and pseudo-just characteristics of the state; those features of the state which were irrelevant for the invention of writing and bookkeeping remained outside the perspective of writing. In this respect, ED Sumer was a *dual society*, one of whose faces was still "mathematical".

From c. 2350 to c. 2200 BCE, the Sumerian area (and soon the whole of Mesopotamia and even more) was united into a single territorial state; after a short-lived centralization around a Sumerian city-king, the centre was the Akkadian "Sargonic" state.

"Literature", originally apparently a free creation of the scribe school and a means for the scribes to probe and demonstrate their professional identity, was soon taken over by the Sargonic rulers as propaganda. While mathematical administration certainly expanded [Foster 1982], the use of supra-utilitarian problems in mathematics teaching was continued; there is no reason to presume that they fulfilled, or could fulfil, any role outside the school.

Already during the ED phase (documented in ED III) but accelerating during the Sargonic period, metrologies were adjusted with concern for mathematical regularity as well as administrative convenience. The former concern (mathematical regularity) is especially visible in the weight system, apparently a fresh development of the ED phase, where the step factor 60 was given a prominent position (only one factor had to be 3×60 in order to accommodate "natural" measures). But other metrologies too were extended upwards and downwards with this factor.

The latter concern (administrative convenience), at times but not always in conflict with the former, asked for the adaptation to administrative procedures or technical practice, for instance in the definition of a Sargonic "royal" gur (the largest capacity unit) and in the creation of particular brick metrologies geared to the various standard bricks.

All in all, the relation between the state and its mathematics seems to have developed during the later ED and the Sargonic period along lines known from other states provided with an accounting or otherwise mathematically organized administration: mathematics was taught in a way which was needed by future staff, but it was also allowed a certain autonomy in the school. It was certainly not taught by "mathematicians" – but even when teachers are supposed to teach for practice, teaching will normally be affected by the fact that the practice with which teachers are *really* familiar is the practice of teaching. Thus also here, according to the meagre evidence at our disposition.

The Janus-faced innovations in metrology corresponds to this tension in the situation of mathematics: sexagesimalization is likely to have been driven by a preference for intra-mathematical coherence, the other innovations by the links to extra-mathematical practice, in particular in the administrative procedures of the state.

The Neo-Sumerian state

Around 2200, the Akkadian territorial state lost most – in the end all – of its territory, and smaller states were reestablished, of which only Gudea's Lagaš (2141–2122 BCE) has left sources that might be considered relevant for our topic – inscriptions telling in meticulous accounting what he has given to the temple, and how he laid out the geometrical plan for sacred buildings. From 2112 BCE onward, however, the Third Dynasty of Ur established a new "Neo-Sumerian" territorial state or empire, mostly referred to as "Ur III".

The early decades of this dynasty present us with nothing spectacular. In 2074 BCE, however, king Šulgi undertook a military reform, which was immediately followed by an administrative reform. From this point onward and until the collapse of the empire, hundreds of thousands of accounting tablets inform us about the details of the administration (and, indirectly, about its governing principles).

At least in the Sumerian South, the larger part if not the overwhelming majority of the working population in both agriculture and handicraft production seems to have been submitted to conditions close to those of slavery,[11] working in crews under scribal overseers who were responsible for the work performed in units corresponding to $^{1}/_{60}$ of a working day (i.e., 12 minutes).

The accounts of the overseers are extremely meticulous, converting all outputs into a common unit,[12] taking illness, death and absence as well as workers lent to or borrowed from other overseers into account. The old administrative calendar was still in use – Ur III is the epoch in which sources show that the overseer scribes were to press out of their crew 30 days' work each month irrespective of its actual length.

As shown by Robert Englund, the yearly deficits of an overseer scribe were accumulated, and at his death the family was held responsible for it (if needed by being drawn into the enslaved crews)[13] – at least in private discussion, Englund would speak of the system as a *Kapo* economy.

For use in this immensely expanded accounting, two decisive mathematical innovations appear to have been introduced.

One is the accounting system itself, with built-in automatic controls (in this respect an analogue of what was brought about in the later Middle Ages by the introduction of double-entry bookkeeping). This was taken over in the subsequent "Old Babylonian" period, during which it was also used for private large-scale accounting – after which it was forgotten.

The other was the sexagesimal place-value system. As is well-known, this was a floating-point system, serving equally well for integers and for fractions. It served for intermediate calculations (of which relatively few traces remain), in mathematical school texts (where orders of magnitude could be presupposed, could be remembered, or were immaterial) and in the late astronomical tables, where the tabular format helped to determine orders of magnitude.

Neither school texts nor astronomical tables can have been the original purpose for which the system was introduced – the latter already for chronological reasons. Nor did it ease additive and subtractive computations (which anyhow were apparently performed on some abacus-like device [Høyrup 2000a]). What it did facilitate was multiplication and division – but only if multiplication tables and tables of reciprocal

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[11] A survey of the debate about how to interpret the sources on this account will be found in [Englund 1990: 63–68]. The system seems to have been established during what seems originally to have been a state of emergency declared at the same occasion as the military reform and which was son made permanent [*ibid*, p. 57].

[12] Often silver, but barley was another possibility – see [Englund 1990: 18–20].

[13] See [Englund 1990: 46*f* and *passim*].

numbers were available or learned by heart, along with tables permitting the translation of metrological units into sexagesimal multiples of a standard unit.[14] The production and teaching of such tables, on the other hand, had no point before the place-value system was in use.

This observation leads to a striking conclusion: The important step was not the *invention* of the new notation – which, by the way, was in the air since centuries, as shown by Marvin Powell [1976], and may even have been invented well before Ur III without leaving any traces in tablets that happen to have survived and to have been read by Assyriologists. What was decisive will have been a *political decision to implement it* – a decision which could only be effectual in a centralized system like Ur III.

We have no direct evidence for the taking of such a decision nor for where it was taken;[15] but we may safely assume that the planning was made in a scribe school environment that was closely connected to the royal administration. Similarly, F. R. Kraus [1973: 24–27] concludes that official year names, royal inscriptions and royal hymns were produced in the subsequent Old Babylonian period in an institution which at one and the same time served as "palace school" and as "court chancery", and that this institution went back to some similar Ur III institution.

That king Šulgi himself (or at least those who produced propaganda in his name)

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[14] I borrow the following explanatory example from [Høyrup 2001a: 18]:

> If [...] a platform had to be built to a certain height and covered by bricks and bitumen, a "metrological table" had to be used to transform the different units of length into sexagesimal multiples of the nindan and kùš, allowing the determination of the surface and the volume in the basic units sar and [volume] sar. A list of "constant coefficients" (igi.gub) would give the amount of earth carried by a worker in a day over a particular distance, the number of bricks to an area or volume unit, and the volume of bitumen needed per area unit – all expressed in basic units (if no transformation into basic units had taken place, different coefficients for the bitumen would have had to be used for small platforms whose dimensions were measured in kùš and for large ones measured in nindan). With these values at hand the number of bricks and the amount of bitumen as well as the number of man-days required for the construction could be found by means of sexagesimal multiplications and divisions – once again facilitated by recourse to tables, this time tables of multiplication and of reciprocal values. Finally, renewed use of metrological tables would allow the calculator to translate the results of the calculations into the units used in technical practice.

The kùš is $^{1}/_{12}$ of a nindan, the sar is one square nindan. A volume sar is a prism with base 1 sar and height 1 kuš.

[15] Until this very year, direct evidence for use of the notation during Ur III was itself extremely scarce (and not fully compelling), in particular because of the uncertainty of palaeographic dating of tablets containing only numbers (that is, of mathematical tables and scratch pads for computation). Now, however, Eleanor Robson [personal communication] has discovered tables of reciprocals found in dated contexts.

saw the school as an essential tool for his project is obvious from one of the so-called Šulgi hymns,[16] according to which the king was taught from an early age in the "tablet-house", learning the art of writing together with addition, subtraction, counting and accounting under the protection of the scribal goddess Nisaba; later we are told that his praise is song in the same tablet house.

Considering the marvellous feats of which Šulgi boasts elsewhere in this and other hymns we may wonder at the level of the mathematical curriculum, far below the actual level of mathematical competence of which the texts of the Old Babylonian age bear witness – even multiplication goes unmentioned, at most it is presupposed as an auxiliary technique in accounting (but why then mention addition?). Actually, however, this fits what can be derived from the absence of all mathematical school texts apart from model documents, in particular when viewed in the light of evidence offered by the terminology of the Old Babylonian period. It appears that *problems*, well represented in the (meagre) corpus of mathematical texts surviving from ED III and the Sargonic period, were banished from the Ur III school: even the modicum of independent thought that is presupposed when the student has to find and not just follow a prescribed way seems to have been considered a threat to their docility.[17]

If any ruler ever *was* the state, the deified Šulgi certainly was. The various Šulgi hymns and the prologue of the law-code he produced[18] are therefore informative about the official ideology of the state. Šulgi is not only a potent military leader and pitiless avenger of wrongs (which, conveniently, permits him to provide slaves) but also a "good shepherd" and exceedingly just (dual society, passed away in late ED III, had not been resurrected). However, only one feature of his justice goes beyond the trite commonplaces of the preceding centuries, and only one thus rings true: metrological reform.

All in all, Ur III repeats features which already appeared to characterize proto-literate Uruk: the management of the state was meticulously planned and controlled, which had several effects:

– In mathematics, important innovations were introduced – one of them with lasting importance, given that the sexagesimal place-value system may possibly have provided part of the inspiration for the Indian introduction of the decimal place-value system and was certainly the direct inspiration for the introduction of decimal fractions. Free supra-utilitarian developments, on the other hand, appear

[16] Šulgi-Hymn B, l. 13–19 [ed. Castellino 1972: 31*f*]. Castellino's translation and commentary misses the mathematical points.

[17] The full argument for this in unfolded in [Høyrup 2001b].

[18] At first ascribed by Assyriologists to his father Ur-Nammu and hence known as the Ur-Nammu laws. The law-code is published with a translation in [Finkelstein 1969], hymns B and C in [Castellino 1972], A, D and X in [Klein 1981].

to have been blocked.

– Socially and ideologically, the fact that the extremely oppressive policies of the system were metered out according to mathematical rules, permitted that these could be seen by those in power – and probably even by the overseer scribes – as exponents of *justice*.

Workers, however, fell ill or ran away the best they could – even this can be read from the accounting texts; after all, they had not been brought up in the scribal school and may have had other opinions about justice if at all caring about such questions.[19] This is likely to be one of the reasons that the Ur III state did not outlast the third millennium. All in all, this early instance of immoderate Taylorism seems to have provoked a reaction similar to what British trade union activist of the twentieth century CE responded to the "scientific management" of their own days: "time and motion studies means that motion stops and time is wasted". Or, in different words which Ur III shows to be equally addressed to those who believe too blindly in the possibility to make mathematization prescriptive:

Ja, mach nur einen Plan
sei nur ein großes Licht!
Und mach dann noch 'nen andern Plan
Gehn tun sie beide nicht

[19] Evidence exists that some of them did. An Old Babylonian epic which appears to reflect Ur III and not Old Babylonian conditions (*Atra-ḫasīs*, ed., trans. [Lambert & Millard 1969]) transposes a strike into the realm of the gods. After the creation of the world, An took possession of the heavens, Enlil of the earth, and Enkidu of the waters below the earth – and the minor gods were put to work, digging Euphrates and Tigris. After toiling for forty years they revolt, set fire to their spades and prepare an attack on Enlil's abode. So much in the account reflects the psychology of real wildcat strikes (Enlil asking who is the instigator, the mutinees answering that everyone is the instigator) that we may safely assume that the story builds on historical experience.

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