Introduction to propositional logic

Torben Braüner

Roskilde University

Plan

- Symbolization of sentences
- Syntax
- Semantics

In propositional logic we consider declarative sentences

Examples

”The sun is red”

”All sparrows are birds”

”My name is Torben”

”It is raining”

Note: There are other sorts of sentences

Syntax of propositional logic

The propositional symbols `p`, `q`, `r`, ... stands for propositions

Propositional symbols are atomic formulas with which compound formulas are built using the connectives

$$\neg, \ \wedge, \ \vee, \ \Rightarrow$$

together with parantheses

The connectives stand for respectively

”not”, ”and”, ”or”, ”implies”

Examples of formulas

$\neg p$, $(\neg p \vee q)$, $(p \Rightarrow q)$, $(((p \Rightarrow q) \wedge p) \Rightarrow q)$

The formula $\neg p$ is built using the connective $\neg$ and the atomic formula p,

The formula $(\neg p \vee q)$ is built using the connective $\vee$ and the formulas $\neg p$ and q, etc.

Parantheses are often omitted like in the formula $\neg p \vee q$

Deklarative sentences are <u>symbolized</u> using formulas

Examples of symbolizations

If $\mathtt{p}$ and $\mathtt{q}$ symbolizes the sentences

”The sun is red” and ”My name is Torben”

Then the formula $\neg\mathtt{p}$ symbolizes the sentence

”The sun is not red”

and $\neg\mathtt{p}\vee\mathtt{q}$ symbolizes

”The sun is not red or my name is Torben”

Semantics of propositional logic

We call `T` og `F` <u>truth-values</u>

To <u>assign</u> a propositional symbol the truth-value `T` is to assume that it stands for a true proposition

Analogously, to assign a propositional symbol the truth-value `F` is to assume that it stands for a false proposition

A propositional symbol is assigned either `T` or `F`

Truth-tables

To each connective there is a <u>truth-table</u>

φ	$\neg\varphi$
T	F
F	T

φ	ψ	$\varphi\wedge\psi$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$\varphi\vee\psi$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$\varphi\Rightarrow\psi$
T	T	T
T	F	F
F	T	T
F	F	T

Thereby any compound formula can be given a truth-table

(Greek letters φ, ψ, θ, … stand for arbitrary formulas)

Examples of truth-tables

p	$\neg p$	$\neg\neg p$
T	F	T
F	T	F

p	$\neg p$	$p \lor \neg p$
T	F	T
F	T	T

p	q	$\neg p$	$\neg p \lor q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

p	q	$\neg q$	$p \land \neg q$	$\neg(p \land \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

Alternative to truth-tables

$\neg\varphi$ is true if and only if φ is not true

$\varphi\wedge\psi$ is true if and only if φ is true and ψ is true

$\varphi\vee\psi$ is true if and only if φ is true or ψ is true

$\varphi\Rightarrow\psi$ is true if and only if φ is true imples that ψ is true

Such <u>truth-conditions</u> contain the same information as the truth-tables

A couple of definitions

A formula is a <u>tautology</u> if and only if it is true whatever truth-values the involved propositional symbols are assigned

Two formulas are <u>logically equivalent</u> if and only if they have the same truth-table

(That is, the formulas φ and ψ are logically equivalent if and only if the formula $(\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi)$ is a tautology)

Examples

p is equivalent to $\neg\neg p$

$p \vee \neg p$ is a tautology

$\neg p \vee q$ is equivalent to $\neg(p \wedge \neg q)$
(and also to $p \Rightarrow q$)

(Se earlier slides)

A tautology is true exclusively due to its form

Linguistically, a tautology corresponds to a sentence that is true exclusively due to its grammatical form

Examples

If r symbolizes the sentence

”The sun is shining”

then $r \vee \neg r$ symbolizes

”The sun is shining or the sun is not shining”

Such a sentence does not give any information

Another definition

The formula φ is a <u>logical consequence</u> of the formulas ψ_1, ..., ψ_m if and only if φ is true for all assignments of truth-values where ψ_1, ..., ψ_m are all true

(That is, if and only if the formula $(\psi_1 \wedge ... \wedge \psi_m) \Rightarrow \varphi$ is a tautology)