Uncertainty modelled using Probability, applications of Bayes formula for conditional prob.

Plus: Solutions to exercises from last time

Written assignments (!)

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this version with small corrections 20 oct 2005.

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Program of today

- Presentation of solutions to exercises 3 and 5 from last time
- Probability: Short intro and working in groups with exercises
- Written assignments, we need to get everyone started!

Exercises from last time

Ex. 1. Robots walking in 2D space.

```
trip(To) --> trip(point(0,0),To).

trip(Here,Here) --> [].

trip(point(FromX,FromY),To) -->
   step(vector(DeltaX,DeltaY)),
   { NewFromX is FromX + DeltaX,
     NewFromY is FromY + DeltaY },
   trip(point(NewFromX,NewFromY),To).

step(vector(1,0))  --> [forward].
step(vector(-1,0)) --> [back].
step(vector(0,1))  --> [up].
step(vector(0,-1)) --> [down].
step(vector(0,0))  --> [think].
```

Ex 5. Pronoun resolution using assumptions

```
discourse --> ss.
ss     --> [].
ss     --> s, ss.

s    --> np(_,Who), [shoots], np(_,Whom),
         {event(shooting,Who,Whom)}.

np(Gender,Who)    --> pro(Gender,Who),
np(Gender,Who)    --> name(Gender,Who),

name(masc,luckyLuke)       --> [luckyLuke].    % ...
name(fem,calamityJane)     --> [calamityJane]. % ...

pro(masc, Who) -->    [he],
pro(fem,  Who) -->   [she],
pro(masc, Whom)   --> [him],
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pro(masc, Who) -->    [he],   {-acting(masc,Who)}.
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- (there are other, more or less ad-hoc weighting mechanisms applied in expert systems, etc. ...)

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- Important: Probability function P(V=...) is a ***mathematical definition***, which has nothing to do with "average of ..."
- However: Probabilities should reflect reality, e.g., be defined from statistics.... which is a different matter!

Operations on events: $\cup$ and $\cap$

Corresponds to set operations on events

$P(V=x_1 \cup V=x_2) = P(V=x_1)+p(V=x_2)$ if $x_1 \neq x_2$

$P(V=x_1 \cap V=x_2) = 0$ if $x_1 \neq x_2$, ... not interesting

More interesting when applied for different rand. var's

$P(V=x \cap W=y)$

requires joint distribution given (as math. def.).

We could (but do not) write as $P(VW=(x,y))$.

Important: No a priori relationship

$P(V=x \cap W=y) = P(V=x)$???? $P(W=y)$

Conditional probabilities

Informally P(A|B) means probability of event A given that B has occurred (been observed)

Example: P(red-haired | girl) which abbreviates
P(V=red-haired | W=girl)
where possible values are W∈{boy,girl}, V ∈ {...}

Definition:

$$P(A|B) = P(A \cap B) / P(B)$$

Fits with intuition of Prob ≈ Relative Frequency:

$$P(rh \mid g) \approx \big(\#(rh \cap g) / \#(b \cup g)\big) \big/ \big(\#g / \#(b \cup g)\big)$$
$$= \#(rg \cap g) / \#g$$

Dependent and independent events

Definition: Random variables V and W are *indep't* if
$P(V=x \cap W=y) = P(V=x) \times P(W=y)$ for all x, y

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Example: What do the following mean intuitively?

P(red-haired ∩ girl) = P(red-haired) × P(girl)

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Example: What do the following mean intuitively?

P(red-haired ∩ girl) = P(red-haired) × P(girl)

P(red-haired | girl) = P(red-haired)

Definition: Two random variables are *dependent* if the are not independent ;-)

Now you do the work

Exercises in section 2.1 + 2.2 of the note "Examples and exercises for conditional probabilities and Bayesian reasoning"

If you have no done it already, start reading text of section 2.1

NB: Notice also new concept of "exhaustive set of events" and Bayes' formula (3.11), plus sum versions 3.12–13.

Bayesian reasoning

Bayes' formula: Twisting conditional probabilities

$$P(A \mid B) = \frac{P(A \mid B) \times P(A)}{P(B)}$$

Correction: Swap A and B here • and here •.

Splitting up p(B) in two cases, conditioned with A and ¬A:

$$P(A \mid B) = \frac{P(A \mid B) \times P(A)}{P(B \mid A) \times P(A) + P(B \mid \neg A) \times P(\neg A)}$$

Example with A=woman, B=read-haired ....

An example ...

A red-haired person is seen running away from scene of crime...

Police has two suspects in custody, both red-haired, a man and a woman.

Who did it (probably):

$$P(\text{woman}|\text{red}) = \frac{P(\text{red}|\text{woman}) \times P(\text{woman})}{P(\text{red}|\text{woman}) \times P(\text{woman}) + P(\text{red}|\text{man}) \times P(\text{man})}$$

Well, if we know P(man), P(woman), how many men typically are red-haired and do. for women...

Some definitions and clarification: ...

Prior and posteriory probabilities

Correction: Swap A and B here .

$$P(A \mid B) = \frac{P(A \mid B) \times P(A)}{P(B \mid A) \times P(A) + P(B \mid \neg A) \times P(\neg A)}$$

Probabilities P(A) and P(¬A) are called ***prior*** probabilities as they refer to probabilites that are given before any event has been observed

Probability P(A|B) and P(¬A|B) are called ***posteriory*** probabilities as they are calculated after event has been observed

Back to the example ...

Consider again:

$$P(\text{woman}|\text{red}) = \frac{P(\text{red}|\text{woman}) \times P(\text{woman})}{P(\text{red}|\text{woman}) \times P(\text{woman}) + P(\text{red}|\text{man}) \times P(\text{man})}$$

We may have P(woman)=0.6 and P(man)=0.4.

But add now "80% of all criminal are men"...
changes P(woman)=0.2 and P(man)=0.8, so
with new prior probabilities, new posteriori are calculated...

Now you do the work

Exercises in section 3 of the note
"Examples and exercises for conditional probabilities and Bayesian reasoning"

If you have no done it already, start reading text of section 3

1. Work with the red-haired woman/man example
2. More natural example about medical tests

On written assignments (see docu on course web)

- Work in groups of 1-3 persons
- Make apointment with teacher about who and what
- Prepare
  - report of. say, 4-7 pages, latest 10 nov
  - short presentation for fellow student, 15 nov
- Report must be approved by teacher
  - let us discuss problems, progress and preliminary versions fo report, as...
  - there is no time for reject-rewrite-approve
- Warning: Level of ambition ...

Possible topics (see note for details)

Should be related to course material

- may be extended exercise or "project" mentioned in earlier notes
- may be a topic you find interesting or have some background
- implementation+perspectives or overview

Examples

- Abductive reasoning
- Language analysis
- Fuzzy logic/fuzzy expert systems
- Bayes reasoning/Bayes network and learning
- Neural nets, an application with specific system
- Evolutionary computing
- ....