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A New Art in Ancient Clothes

Itineraries chosen between Scholasticism and Baroque in order to make *algebra* appear legitimate, and their impact on the substance of the discipline

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Contents

I. “Local reading” and “global reading”

Different readers approach the same text differently – this is trivially well-known and the *raison-d’être* for the conceptual distinction between “the text” and “the reading”. More often forgotten though almost as trivial is that even the single reader receives messages at several levels at a time when appropriating a text. On one hand, there is the appropriation of the intended message (with due reservation for all the ambiguities involved in this process) – we may speak of the “local reading”, namely of the particular text; on the other hand, the text informs indirectly about the kind of discourse within which it belongs – we may speak of the “global reading”, namely of a *reading of the discursive space* within which the text belongs through its reflection in the particular text.

As far as texts belonging within a familiar discursive space are concerned, the immediate role of the global reading is to provide invisible confirmation and maintenance of the prevailing discursive order. This order, however, is never static: since a discursive space only exists as the totality of the discourse it contains, its stability can only be that of a dynamic equilibrium; its gradual adaptation to changing contents mostly takes place by way of the invisible maintenance through unnoticed global readings, i.e., through acceptance of actual texts which change the underlying presuppositions of the discourse (presuppositions about legitimate themes and styles, about legitimate argument types, about *topoi* and their function, about the contents of common notions, etc.).

When texts belonging to a less familiar discursive space are read, the situation is different. In this case, understanding of the originally intended message depends on the ability of the reader to appropriate the underlying presuppositions, which cannot be taken for granted.

In a first approach, that appropriation is likely to quite deficient. The deficiency involved is to be distinguished from the dilemma which inevitably presents itself whenever one tries to understand a discursive space *S* from the stance of another space *T* to which the participants of *S* had no access: understanding from without by necessity involves that the concepts (etc.) of *S* are seen through, in relation to or in contrast to those of *T*; it implies a process of relativization which is absent when *S* is understood *solely* from within. The imperfection of a first approach, however, involves a very different and much more elementary problem: namely the inability to reconstruct / follow the internal connections between and the mutual conditioning of the constituents of space *S*. Moreover, since the formulation of the direct message of a text assumes the presuppositions of its discursive space, imperfect appropriation of these entails that even the "local" understanding of the contents of the texts will be deficient, misconstrued or outright wrong.

These preliminary considerations, including the concepts of "local" and "global" readings and the notations of spaces *S* and *T*, will serve in what follows. Simplifying as they are, they constitute a framework within which the fuzzy picture presented by actual historical texts can be distended.

II. Algebra and the discourse of ancient mathematics

Much of the activity of "mathematicians" between *c.* 1000 CE and 1700 CE involved efforts to learn to make mathematics "in the way of the ancients" or to show that what was made belonged indeed within the framework defined by the ancients.[1]

In "classical" fields like geometry – that is, fields which according to their name and a broad description of their subject-matter were continuations of ancient mathematical disciplines – this effort took shape as a struggle with the direct messages of the ancient texts; penetration of these would gradually accumulate comprehension of the underlying presuppositions. What went on in such cases is thus described by the "Hermeneutic circle" in Schleiermacher-Dilthey interpretation: the mutual elucidation of the totality of a textual universe and its single constituents or details.

[1] These time limits are of course somewhat arbitrary. As boundary posts I have chosen the attempts of a handful of early eleventh-century scholars to understand the concepts occurring in the few available ancient texts – see [Tannery 1922: 86–93, 103–111, 229–303]; and the final acceptance of *algebraic computation* (including *analysis infinitorum*) as a legitimate tool *sui generis* for the mathematization of nature and mechanics in the early eighteenth century.

At the same time, however, it illustrates why Heidegger and Gadamer would change this interpretation of the term: as Renaissance and Early Modern mathematicians became familiar with the ancient mathematical project and appropriated its presuppositions ("space *S*"), they came to understand more about their own project ("space *T*"); but the outcome of the process was (and could only be) a new project, connected to the wholly different social references and epistemological tasks of Early Modern mathematics.[2]

The situation is different in the case of algebra. This discipline (better, this "art") was taken over from the Islamic world, mainly in al-Khwārizmī's early version, where Greek influence was only marginally present in the lettering of diagrams in his geometrical proofs.[3] Whether defined in terms of the quadrivium or according to Aristotle's less restrictive notion of "subordinate sciences" (that allowed optics and statics to be regarded as mathematical disciplines), *al-jabr* / algebra remained a strange bird.

This situation could be handled in one of three ways:

(i) Algebra could, tacitly or explicitly, be considered to fall outside the scope of what mathematics should be, and thus be disregarded; such ideals about mathematics, though accompanied by little mathematical substance, had been transmitted by Martianus Capella and later handbook authors. However, I know of no instances of explicit rejection; whether mathematical writers who did not touch or refer to algebra acted so because of tacit disregard or simply because they were interested in or needed other disciplines is probably undecidable – at least I am not able to point to any mathematician between 1100 and 1700 who "should" have made use of algebra (i.e., who might profitably have made use of the technique) and

[2] Below (p. 29) we shall see how the characteristics of the new project manifests itself in Bombelli's reinterpretation of that preference of "theory" over "practice" which he has taken over from Diophantos.

[3] It is true that Abū Kāmil's *Algebra* was also translated, possibly by Guglielmo de Lunis [Sesiano 1993: 322f], and that some references to *Elements* II and VI are found in this work. However, the translation in question seems to have had virtually no influence; moreover, the total of 11 Euclidean references certainly do not counterbalance the overall stylistic impression, which is definitely non-Greek.

knew so but abstained deliberately from doing it.[4]

The closest we can come at rejection seems to be Abraham bar Hiyya's (Savasorda's) Hebrew *Collection on Mensuration and Partition*. This Hebrew treatise, written in the border region between the Islamic and the Latin worlds, contains a number of quasi-algebraic problems which are solved by means of the techniques of *Elements* II and without reference to *al-jabr*. Non-use is not to be confused with avoidance, however, and at closer inspection Abraham turns out to have had good reasons beyond possible considerations of legitimacy for not making such references. Firstly, they would probably not have been of much help to his target audience: The Jewish community of Provence. Next, Savasorda's quasi-algebraic problems are already geometric, and there was no reason that Savasorda should translate them into *al-jabr* problems about amounts of money and their square roots. The geometric methods he uses are indeed much closer to the methods by which these same problems had been solved for some 3000 years – methods which appear both to have inspired *Elements* II.1–10 and al-Khwārizmī's geometrical demonstrations [cf. Høyrup 1993]. Nothing in Abraham's text calls with any kind of necessity for the use of *al-jabr* – *pace* Levey [1970: 22] who sees the work as "the earliest exposition of Arab algebra written in Europe".

(ii) The situation might be accepted without being perceived as a dilemma – many writers accepted algebra as a mathematical discipline or technique without bothering about any failing agreement with the norms of ancient mathematics.

(iii) Finally, attempts could be made to show that algebra was actually part of ancient mathematics, or to transform it in a way that made it agree with ancient norms. The disparate way various writers did so reveals different conceptions of the criteria that would allow to identify a branch

[4] I should say that I am not broadly familiar with seventeenth-century sources. On the other hand, competent mathematicians who disregarded algebra in the seventeenth century had to disregard Viète (and, after some decades, Descartes), i.e., a discipline that had already been fully appropriated by modern mathematics (though not for all purposes where later mathematics finds it appropriate – cf. [Bos 1993]). This is already a different matter and not very relevant for the present inquiry. The one seventeenth-century example to be discussed in some detail below (Caramuel) becomes relevant because he chooses *not* to follow the lead of the "competent mathematicians".

of mathematics as ancient or as legitimate according to ancient norms – in other words, to different global readings of the discursive space of ancient mathematics (or ancient letters in general). These different readings will be the main topic of what follows; in order to elucidate the background on which these redefinitions of algebra took place, examples of the accepting attitude will occur in between.

The gradually changing foundations of the discursive space (rather, spaces) of proto-Modern and early Modern "mathematicians" reflect the global readings of ancient mathematics undertaken by its participants; but a discursive space never reads on its own, and only reflects the readings of participants in mediated form. This is a major reason that the inquiry focuses on individual writers though seen in the context of the communities and discursive spaces to which they belonged (other reasons have to do with the relationship between the relatively small communities of which we speak and their individual participants, and with the character of these communities – problems that will be briefly addressed in note 42).

III. Twelfth-century reception

The first Latin work presenting the basics of algebra may be John of Seville's *Liber Alghoarismi de pratica arismetrice* [ed. Boncompagni 1857a: 23–136]. Pp. 112–113[5] contains excerpts from "the book called *gleba mutabilia*",[6] and simply tells the three basic algorisms for solving mixed second-degree problems, illustrated with the examples $r+10\sqrt{r} = 39$, $r+9 = 6\sqrt{r}$ and $3\sqrt{r}+4 = r$ (r stands for *res* and $\sqrt{r}$ for *radix sua*); there are no geometric demonstrations, and neither references to the origin of the technique (apart from the unfamiliar name) nor any attempt to connect it to familiar mathematical concepts (apart from its treatment in a book on the "practice of arithmetic"). The work as a whole, it is true, locates its subject-matter within the familiar framework: The prologue has a clearly

[5] Here and in what follows, unidentified page numbers refer to the edition of the source text which has been indicated previously.

[6] Here as elsewhere, translations from original languages are mine when nothing else is stated.

Boethian ring, reminding of *De institutione arithmetica* I.II, even though only the definition "numerus est unitatum collectio" is taken over word for word (from I.III); towards the end (p. 128), the fact that there are only 9 "primary numbers" (1, 2, ..., 9) is explained with reference to evidently Christian numerology (God is ternary).

Another possible "first" is Robert of Chester's translation of al-Khwārizmī's *Algebra* [ed. Hughes 1989], dated 1183 Spanish era, i.e., 1145 CE. Even though this work is a full presentation of the art, the attitude to the "foreign" subject and its relation to the familiar understanding of mathematics is no different – or, if different, Robert points more directly to the Arabic origin: Muḥammad ibn Mūsā al-Khwārizmī ("Mahumed filius Moysi Algaurizmi") appears in the initial lines, both as author and as worshipper of God the Creator who brought man the "science of knowing the force of numbers"; and indubitable Arabic loanwords identified as Arabic surface as technical vocabulary in the discussion of the rule of three (p. 65). At the same time, an added phrase in the beginning, evidently taken over from a Latin translation of *Elements* VII, def. 1, tells unity to be "that by which every thing is said to be one", "qua unaqueque res dicitur una".[7] Nothing is done to veil the Arabic origin of the discipline, but a slight addition connects it to familiar knowledge. As to the actual contents of the art that is taught, Robert renders his Arabic text faithfully as far as it goes. The chapters on mensuration and legacies are absent and appear to have been so from the Arabic manuscript – as Robert says in the concluding remarks (p. 66), "beyond this there is nothing more".

Slightly later in date is Gerard of Cremona's translation [ed. Hughes 1986]. It is, as Gerard's translations in general, extremely faithful to the original – as I have discussed elsewhere [Høyrup 1991], Gerard's text is a better witness of al-Khwārizmī's original wording than the published Arabic manuscript text – again as far as it goes, since even Gerard's Arabic manuscript breaks off after the chapter on the rule of three (p. 257: "here the book ends"). There are no traces of for instance Boethian rhetoric. The

[7] The formulation in Adelard I [ed. Busard 1983] and the Hermann translation [ed. Busard 1972: 139] only differs by having "omnis" instead of "unaqueque"; "Adelard II", possibly stemming from Robert's own hand, furthermore has "ex qua" instead of "qua" [ed. Busard & Folkerts 1992: 187].

only compromises with traditional understanding of mathematics is, firstly, the use of current mathematical terminology, which makes Gerard use the same Latin term (e.g., *aggregare*) to render several Arabic terms (in case, *balaġa*, "to reach", *jamaᶜa*, "to gather", and *ajtamaᶜa*, "to be / come together" – see [Høyrup 1991: 25]); secondly, the elimination of the initial praise of God and the Caliph.

Gerard's translation of the *Liber mensurationum* [ed. Busard 1968], in which *al-jabr* (referred to as *aliabra*) is used as a tool, is similar in character in as far as the *al-jabr* sections are concerned, and thus does not change the overall picture. Nor is this picture changed by Leonardo Fibonacci's *Liber abaci* [ed. Boncompagni 1857] and *Pratica geometrie* [ed. Boncompagni 1862], with the exceptions that Leonardo already borrows freely from the predecessors (Savasorda as well as Gerard[8]) and that the *Pratica* changes the traditional al-Khwārizmīan demonstrations, inserting explicit use of *Elements* II.5–6 (in the *Liber abaci*, implicit use is made of the same theorems); for this he could take inspiration from Savasorda as well as Abū Kāmil, whose algebra he will also have known either directly or indirectly [Vogel 1971: 611]; this simply brought the references to the familiar up to date. The *Liber abaci*, it barely needs mentioning, also investigates a much larger range of problems than the predecessors.

Algebra, as known in early thirteenth-century Latin Europe, thus confessed its Arabic origin without difficulty; apart from obvious spelling problems, it had no difficulty with the presence of Arabic loanwords and names. It was a rhetorical algebra, never organized in a deductive structure; it was reasoned, in the sense that the rhetorical reduction of problems was its own justification, and that the algorithms for solving reduced problems were argued with reference to geometric diagrams (mostly proofs in al-Khwārizmī's style, at times connected to *Elements* II). And it was built up around arrays of single numerical problems – *experimenta*, in the term used

[8] The translation of Arabic *māl* as *census* is probably borrowed from Gerard: Robert uses *substantia*; John of Seville has *res*, implying that his source speaks merely of *šay'*, "the thing", and its root. (Cf. the more detailed arguments for Leonardo's use of Gerard in [Miura 1981]). Leonardo's occasional use of *avere* for *māl* is an intriguing hint that he may also have drawn on earlier translations from the Arabic into the vernacular – translations of which all direct traces seem to be lost. If this is indeed so, then the algebra tradition of the abacus school may not be derived exclusively from the *Liber abaci*.

by Richard de Fournival (*Biblionomia*, nº 45, ed. [Birkenmajer 1970 / 1922]).

IV. Jordanus and his *De numeris datis*

In the same catalogue, Richard refers to one of Jordanus's works as an "apodixis super practica que dicitur algorismus" (nº 45); Jordanus himself uses the corresponding Latin term "demonstratio"; soon the latter characteristic was used by contemporaries to characterize even Jordanus's *De elementis arithmetice artis*, even though Jordanus and his closer associates appear not to have employed it.[9]

These terms and their use are important for understanding the Jordanian project. An *apodixis* or *demonstratio* is a treatise on a subject made in agreement with the Aristotelian precepts from *Analytica posteriora* – and, since Aristotle's model is geometry, in the form of the Euclidean *Elements*. There were thus good reasons that the *De elementis arithmetice artis*, following precisely that model, should be considered an *apodixis*, in contrast to Boethius / Nicomachos. Jordanus's treatise *De numeris datis*, on the other hand, was never considered an *apodixis* of its own; it belonged within the global theoretical structure defined by the *De elementis arithmetice artis*, as the original *Data* belong with the theoretical structure based on the *Elements*.

This is of interest because *De numeris datis* was Jordanus's version of algebra. Jordanus does not tell so directly (we shall return presently to the character of the text) – the terms *algebra*, *res* and *census* never occur, and the Arabs only turn up when their particular method for solving the problem of the "purchase of a horse" is presented.[10] But Jordanus's choice

[9] The work which Richard, in agreement with Jordanus's own words, had characterized as *Liber de elementis arychmetice* (nº 47), became *Aritmetica Jordani demonstrata* in the Sorbonne catalogue from 1338, and is likely to have carried this label already at the 1289 reordering of the library ([Birkenmajer 1970 / 1922: 167], cf. [Delisle 1868: II, 181] and [Rouse 1967: 51f]).

On Richard as a member of a "Jordanian Circle", cf. [Høyrup 1988: 343–351].

[10] II.27, [ed. Hughes 1981: 84f]. Without repeating it *verbatim* the text is so close to the corresponding passage in the *Liber abaci* [ed. Boncompagni 1857: 245–243] that a borrowing from Leonardo seems more likely than a common source; the corresponding problem in al-Karajī's *Fakhrī*, to which Hughes points in a note [1981: 184 n.80] differs so much in its choice

of illustrative numerical examples leaves no doubt that he confronts his readers with what *he* wants algebra to be, nor that he wants those readers who are already familiar with customary *algebra et almuchabala* to recognize this aim – see [Høyrup 1988: 335].

Without investigating the work in full detail, we should note the following characteristics:

(i) Like the Euclidean *Data*, the work starts by a set of definitions, not blindly copied but adapted to the arithmetical context.[11] Firstly, of course, purely geometrical references (to position, and to particular geometrical configurations) are eliminated. Secondly, of those concerned with magnitude only the strictly necessary ones are transferred to numbers: a number may "be given", or "be given to another number" (i.e., the proportion is given[12]). Thirdly, even these concepts are defined in a way which fits the particular subject-matter: A number is given if its quantity is known, i.e., if it can be identified numerically; and a proportion if its denomination is known (in arithmetic, no irrational proportions can occur).[13]

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of numbers that it can safely be excluded – see the paraphrase in [Woepcke 1853: 97]. Without being much mistaken, Jordanus thus seems to regard Leonardo as a representative of the Arabic tradition.

Unlike the other propositions, II.27 is only presented as a numerical example; but is follows upon Jordanus's own abstract treatment of the same problem in general form.

[11] I compare with the Latin version of the *Data* [ed. Ito 1980], all known manuscripts of which are connected to Jordanus and his circle [cf. Høyrup 1988: 344].

[12] In order to eschew the understanding of the term "ratio" as a rational number, I shall translate Latin *proportio* as "proportion". The ensuing conflation of ratio and (proper) proportion corresponds perfectly to the usage of Medieval and Early Modern mathematics.

[13] That Jordanus was fully aware of the problem of irrationality is evident from the *De elementis arithmetice artis*, prop. V.12 and V.14. Here Jordanus shows that the division into extreme and mean ratio and the equipartition of a given ratio between numbers can be approximated to any given degree with ratios between numbers [ed. Busard 1991: I, 112–114, cf. 19f]. As observed by Busard, Jordanus "must have been a very good mathematician" if such propositions were of his own making, which they seem to be.

In principle, the "denomination" of a proportion should be its name according to traditional Boethian usage – *dupla* for 4:2, *sesquitertia* for 12:8, etc. In his numerical examples, however, Jordanus uses numbers and fractions when it suits him, and even the general proofs operate on the denominations as with (rational) numbers – thus for instance in II.3, where the denomination of $b{:}a$ is found through division of 1 by the denomination of $a{:}b$. The corresponding understanding of the denomination as the outcome of a division is made explicit by Campanus, and has its roots in the Islamic tradition [cf. Koelblen 1994: 243, 246].

(ii) Next follow the propositions, distributed in four books. Most propositions tell that "if certain arithmetical combinations $C_1(a,b,...)$, $C_2(a,b,...)$, ... are given, then the numbers a, b, ... are also given; others depart slightly from this format without changing its principle. Every proposition is provided with a proof, in which Jordanus makes use of the same letter formalism as the arithmetic and the long version of his algorithms. Finally comes a numerical illustration which, as already said, coincides so often with the one which was familiar in the *al-jabr* tradition or with other well-known Arabic or Latin "recreational" problems that accidental coincidence can be disregarded.

(iii) The propositions are organized in coherent groups – the whole of book I, for instance, deals with two numbers (say, a and b) whose sum or difference is given (in I.2, the sum is divided not into 2 but any number of addends); from I.3 until I.16, Jordanus goes through a large number of cases where the other given number is the value of a second-degree expression in a and b – for instance ab, $ab+(a–b)^2$, $(a+b)\cdot(a–b)+b^2$, $a^2+b^2+(a–b)$, etc. This rather systematic treatment ends up by containing the arithmetical equivalents of a large number of problems from *Elements* II, the *Data*, and the *Liber mensurationum* (and what Leonardo borrowed from this work for his *Pratica geometrie*), and some which to my knowledge are not to be found in any earlier source. Some of these concurrences may be due to deliberate borrowing, while others are likely to be coincidental; in any case, the solutions are modified so that fractions can be avoided (Jordanus uses sums and differences where the three works just mentioned, as also Diophantos, employs semi-sums and semi-differences).[14]

(iv) Book II goes on with problems referring to proportions, beginning with the rule of three (II.1) and ending with pure-number problems more

[14] Particularly illuminating is I.16, which can be translated into modern symbols as $a+b = P$, $ab+(a–b) = Q$. Jordanus's method can be described as a computation of $T = P^2–4Q = (a–b)\cdot(a–b–4)$ (since his numbers are always positive, he has to split in cases). Instead of using the normal method for solving this mixed second-degree problem in $a–b$, Jordanus recurs to factorization of T without telling so too clearly, thus taking advantage of the arithmetical context (disadvantage, as it happens, since the solution is not generalizable). If he had followed the inspiration from the *Liber mensurationum*, he would have proceeded quite differently, adding the two equations and thus finding that $a\cdot(b+2) = P+Q$, $a+(b+2) = P+2$; this would have led him back to his own prop. I.3.

or less close to the "purchase of a horse" (cf. above), while book III is mostly concerned with numbers in continued proportion (including, however, other problems on proportions related to this main group by the techniques used to solve them). Book IV, finally, goes on with a variety of problems of which some have been borrowed from traditional *al-jabr* – not least n^os^ 8–10, the three basic mixed second-degree equations; others appear to be Jordanus's personal extrapolations. The techniques are sometimes identical with those familiar from *al-jabr*, or arithmetical counterparts of geometrical *al-jabr* proofs (thus in II.8–10, which operate by quadratic completion). In other cases Jordanus takes advantage of his results regarding proportions. Thus in IV.19, where for two numbers (a and b) a^2+b^2 and ab are given;[15] ab is observed to be the mean proportional between a^2 and b^2, which reduces the problem to one already solved in III.5 (if, for a:b:c, b and $a+c$ are given, even a and c will be given).

Since the whole of Jordanus's production is oriented toward ancient mathematical ideals, we may take the characteristics of the *De numeris datis* not only as a way to dress up algebra in ancient apparel but also as evidence for his way to read the ancient ideals – ideals which then shaped the way he approached the discipline.

First of all, each mathematical discipline was to be dealt with according to its proper principles; geometry might provide the exemplar for the organization of an *apodixis,* but the actual subject-matter – from definitions and postulates down to single theorems and arguments – had to belong to the discipline itself. And since algebra was a science of finding numbers, algebra was to be dealt with under the heading of arithmetic, and the customary geometrical arguments had to be discarded. At times, we have

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[15] A closely related geometric problem is found in Savasorda's *Liber embadorum* (2.1.18, [ed. Curtze 1902: 48]); in Leonardo's *Pratica geometrie* [ed. Boncompagni 1862: 64]; in the *Liber mensurationum* [ed. Busard 1968: 92]; in the gromatic treatise *Liber podismi* [ed. Bubnov 1899: 511f]; in a Demotic papyrus [ed. Parker 1972: 41–43]; and even in an early Old Babylonian clay tablet [ed. Baqir 1962]: namely, to find the sides α and β of a rectangle with known area and diagonal δ. In all but the *Liber podismi* and the Demotic papyrus, the problem is reduced to one of the standard problems corresponding to *Data*, prop. 84 and 85 (the area $A = \alpha\beta$ and either $\alpha-\beta$ or $\alpha+\beta$ are given); the gromatic/Demotic solution finds $\alpha-\beta$ and $\alpha+\beta$. The Savasorda-Leonardo method will have been known to Jordanus, and the Demotic corresponds to techniques used elsewhere Jordanus's treatise. We may hence conclude that Jordanus's present method reflects a deliberate choice.

seen, Jordanus implements this principle so radically that his proofs cannot be transferred to analogous problems dealing with non-numerical quantity (the factorization in I.16).

This persuasion is of course in agreement with the most orthodox Aristotelianism – "our knowledge of any attribute's connexion with a subject is accidental unless we know that connexion [...] as an inference from basic premisses essential and 'appropriate' to the subject" (*Analytica posteriora* $76^{a}3$–6, transl. [Mure 1928]). Thirteenth-century Aristotelianism, however, was far from orthodox on this account, at least in its way to deal with mathematics – in order to see this we do not need to consider diffuse minds like Roger Bacon, Campanus's didactical commentaries to the beginning of *Elements* V show – in their very effort to establish the difference between number and quantity – that superior disciplines might just as well be explained from subordinate disciplines as vice versa [*Euclidis Megarensis ... Elementorum libri xv*: 103–112]. Since no other feature of Jordanus's writings reflects particular infatuation with Aristotelian philosophy, there is no reason to believe him to have been a more devoted reader of Aristotle than were his overtly Aristotelian contemporaries. Jordanus is therefore more likely to have taken over the principle from his "global reading" of ancient geometry. Even Aristotle's view was of course a generalization of what he had found in geometry, as can be seen from his constant references to geometrical examples when he explains it. The agreement is thus far from accidental.

Also in agreement with Aristotle, and probably for similar reasons, is Jordanus's transformation of a discipline constructed around the solution of specific problems into one dealing with the solubility of generalized problems. *His* was a discipline dealing with necessary truths, and not concerned with the coincidental; that a solution happens to be 5 is no more relevant for science than the coincidence that "'While he was walking it lightened': the lightning was not due to his walking" (Aristotle's example and commentary, *Analytica posteriora* $73^{b}11$–12, trans. [Mure 1928]). In spite of his indubitably Boethian upbringing, Jordanus had learned from what he must have considered "real" ancient mathematics this quest for generality. In order to achieve it he had to produce his famous letter formalism for general number. His actual formulation of this general theory in the format of “givens” was evidently dictated by knowledge of the *Data*.

Finally, *De numeris datis* is more than a reconstruction of *al-jabr*. On one hand, Jordanus uses material from *all* the “quasi-algebraic” techniques known to him – that is, techniques for extricating unknown quantities from complex relationships. Apart from *al-jabr*, the theory of proportions was amply used for that purpose, not least in spherical geometry. Not used in practical computation (nor was, however, second-degree *al-jabr*) but still of quasi-algebraic character was the kind of geometry represented by the Greek technique of application of areas and by the problems on squares and rectangles in the *Liber mensurationum* (with a few analogues in the *Liber embadorum* and many in the *Pratica geometrie*); whether Jordanus took his inspiration from one or the other type (or both) cannot be decided because of his transformation of the material. On the other hand, as it was pointed out, Jordanus erects a theoretical structure of his own, reorganizing, reshaping and supplementing whatever he has borrowed from elsewhere.

To which extent the *integration* of various quasi-algebraic techniques reflects Jordanus’s global reading of ancient mathematics is difficult to tell; having understood their common quality Jordanus may well have decided without reference to ancient models that they belonged together. In any case it corresponds fairly well with the integrative character of the original *Data*. The attempt to reorganize the material in coherent structures, however, goes together with the quest for generality and is no less a reflection of Jordanus’s global reading of that high level of ancient mathematics that had become available shortly before his own times.

Summing up we may say that Jordanus read the underlying global ideals of ancient mathematics so well that he was kept away from all attempts to emulate the specific contents of ancient mathematics directly when making his new version of algebra – be it geometrical arguments, be it Boethian numerical examples and “experiments”.

V. Fourteenth-century interlude

Jordanus was an exception in his times: not only by his outstanding mathematical competence and finesse but also for his lack of manifested interest in scholarly fields beyond mathematics.[16] As a consequence, contemporary and later Medieval writers, even when using his works (making a "local reading", so to speak) would be indifferent to the defining features of his project (omitting hence an attentive "global reading") – cf. [Høyrup 1988: 341–343]. Since, furthermore, *his* version of algebra was utterly different from the habitual technique and mainly of interest precisely because of these defining features, and since the scholarly environment was anyhow not very interested in algebra beyond the rule of three (which turns up in some late Medieval algorism treatises), we should not wonder that the particular message of the *De numeris datis* stayed uninfluential, in spite of a fair number of manuscript copies[17] and some evidence that

[16] Arguments based on Jordanus's lack of interest in non-mathematical matters may seem slippery-after all we know nothing but his mathematical writings, which are very pure in style and therefore seem to leave no space for extra-mathematical references. Yet, even if devoid of biographical information we are not totally deprived of hints of his character. Firstly, the pureness of his mathematics is so atypical that it contributes in itself to his portrait. Secondly, the dedicatory letters and introductions to the presumably early algorisms [ed. Eneström 1907; 1913] show him to be a better Latin stylist than most contemporary scholars, and tell how important he found it to "thread in the footsteps of the ancients" [ed. Eneström 1907: 139]; they also bear witness of a rather sophisticated attitude to the *philosophy* of mathematics, but refer to no scholarly substance beyond the metrologies of Isidore's *Etymologies* and other encyclopediae. Thirdly, the larger part of the *Liber de triangulis Jordani* can with high certainty be identified as a student *reportatio* of lectures held over Jordanus's *Liber philotegni* at a moment when this work was still in progress; the most likely identity of the lecturer in such a situation will evidently have been the master himself, which allows us to guess that the recurrent irony and characteristic style of the treatise belong to Jordanus [see Høyrup 1988: 347–350]. Even in this treatise, no scholarly interest beyond mathematics proper will be found, except for some initial metamathematical definitions which recur in the *Liber philotegni* and correspond to the philosophical attention manifested in the algorisms. All in all, a consistent portrait seems to emerge.

[17] We may notice that the Sorbonne Library volume containing the *De numeris datis* was located in the *parva libreria*, in which duplicates (to which category it did not belong) and works seldom used were found; the *De elementis arithmetice artis*, on the other hand, was in the reference library of chained books in heavy use. See [Birkenmajer 1970 / 1922: 167f] and [Rouse 1967: 43].

it was read occasionally by mathematically interested scholars.[18] Only the young Regiomontanus would point to its “beauty” in his Padua lecture from 1464 [ed. Schmeidler 1972: 46] – but since he describes the *De elementis arithmetice artis* as “excerpted from” *Elements* VII–IX in the preceding sentence, one may wonder whether even he had taken the time for much more than browsing.

Between Jordanus and Regiomontanus, indeed, and with only one noteworthy exception, algebra was thus hardly pursued at all in the scholarly environment. Its continuation and development (to the extent that development occurred) took place instead in the abacus schools of Italian commercial towns. Nothing of what survives from this environment suggests attempts to connect the discipline to ancient mathematics – the main nexus was to the *Liber abaci*, even though other as yet unidentified channels to the Arabic world are likely to have existed. Nor are such suggestions to be found in Chuquet’s *Triparty*.

The noteworthy exception is Jean de Murs, in whose *Quadripartitum numerorum* [ed. l’Huillier 1990] from 1343 algebra takes up considerable space. Jean was certainly no particular friend of the Muslims – definitely less so than most contemporary scholars: in a letter to Clement VI he proposed to take advantage of a favourable conjunction and organize a crusade [Poulle 1973:131]. Roger Bacon and Ramón Lull envisaged similar patriotic applications of their science, but no scholastic mainstream mathematicians appear to have shared this bellicose attitude. Yet when it comes to the substance of his mathematics, Jean does not distinguish between legitimate mathematics, i.e., mathematics in the ancient tradition, and illegitimate mathematics of Arabic inspiration. His algebra is wholly in the tradition established by Gerard and Leonardo[19] even when he

[18] As Cantor [1900: 238] concludes with gentle irony from the production and spread of manuscripts, “hat [*De numeris datis*] gewiss nie gänzlich aufgehört gelesen zu werden, so selten sie auch verstanden worden sein mag”.

[19] L’Huillier [1990: 56] rejects use of “Gerard of Cremona’s version”, but only because she accepts Boncompagni’s identification of the version given in his [1851: 412–435]. As pointed out by Björnbo [1905], anybody familiar with Gerard’s style (which Boncompagni had no occasion to be in 1851) will notice that he could never have made a translation whose initial explanations of number are changed into Boethian style (to mention only what first leaps to the eye when one starts reading). Gerard produced the version which was first published

adjusts its formulation.

An instance of adjustment is found in III.9 (p. 286f). At first we are told that "one is the beginning of multitude, not however *in actu* but potentially, when it forces things together". This almost sounds like the habitual reference to Boethian and similar arithmetical thought, even though the final clause is already suspicious – it seems to mean that 7 cows become *one* entity through being forced together conceptually by the one number 7.[20] More important however is what follows: "Number is one, or one repeated equally any number of times". Instead of offering lip service to ancient mathematical notions, Jean has thus observed that algebra does not suggest any distinction between *one* and *number*; that the definition is meant specifically for algebra follows from its location in the work; in I.1 (p. 147), number is defined in fully traditional way as "a multitude measured by 1, or a multitude brought forth from unities, ..."; the same identification of *number* and *multitude* also runs through the whole numerological prologue as a recurrent theme. Where John of Seville and Robert of Chester would make a minimal compromise with established concepts, and Gerard would just translate the Arabic text faithfully, Jean explicitly introduces a number concept in disagreement with orthodox ways but in agreement with the basis of algebra as inherited from the Arabs.

That no influence from the *De numeris datis* can be traced in the *Quadripartitum numerorum* will therefore come as no surprise. Jean as well as Jordanus aims at mathematical coherence; but whereas this coherence as conceived by Jordanus had to "thread in the footsteps of the ancients", Jean obviously did not bother. Jean could find all the material he needed in traditional treatises; Jordanus's specific metamathematical project did not interest him.

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from a single manuscript by Libri [1838: I, 253–297] and later in critical edition by Hughes [1986]. This version is correctly identified by l'Huillier as the translation on which Jean relies.

[20] L'Huillier refers to a somewhat similar phrase in Jordanus's short algorithm on fractions [ed. Eneström 1913: 42], "as in one whole plurality is found though not *in actu*, thus in the plurality of the divisibles, unity is found potentially". Apart from the standard distinction *in actu* / *in potentia*, however, nothing connects the two statements. The similarity only shows that both authors had grown up within the scholastic environment.

VI. In the shadow of Humanism

Regiomontanus

Even in the fifteenth century, algebra remained in the main an abacus-school interest. In this context of practical or pseudo-practical computation, no attempt was made to disguise the technique in ancient dress. The one important exception to the rule is Regiomontanus. As already mentioned he speaks of algebra in his early Padua lecture; algebraic works are also mentioned in the circular in which he announces his extensive printing plans (1474?), while algebra is used directly in the *De triangulis*[21] and other writings of his; finally, one of the definitions of *De triangulis* may reflect his acquaintance with the *De numeris datis*.

The latter reference, however, is dubious. As we remember, Jordanus had told a proportion to be given if its denomination was *known*; according to Regiomontanus, the proportion is given if even the denomination is *given*, or if both terms of the proportion or of another proportion to which it is equal is known (p. 7).[22] Since Regiomontanus uses "known" and "given" without distinction, a borrowing is not excluded; nor is however independence, since the idea is so close at hand. In view of the generally innovative character of the definitions,[23] however, even a conscious borrowing will have been nothing beyond insignificant lip service and not have implied any sympathy for Jordanus's general interpretation of algebra.

The 1474 circular [ed. Schmeidler 1972: 532] is hardly more informative. What we find there pertinent to our topic are two lines:

> Jordanus's *Arithmetical Elements*. The *Arithmetical data* of the same.
> The *Quadripartitum numerorum*. A work gushing with manifold subtleties.

[21] I used the facsimile of the 1533 Petreius edition in [Hughes 1967].

[22] Equality between proportions is then defined as equality of denominations, which makes the whole thing a bit circular.

[23] Givenness of quantities is defined from numerical measurement, without attention to the problems of incommensurability. Even when writing about geometry, Regiomontanus remained an astronomer who would observe, measure and compute numerically.

Neither work is identified as algebraic, and no other algebraic work occurs in the list. Apart from Vitelo's *Perspectiva* ("an enormous and noble work") and the Campanus version of the *Elements* ("corrected, however, in several places"), moreover, no other writings from the thirteenth and fourteenth centuries appear.

What *can* be concluded is thus the following: (i) Regiomontanus respected these four works highly (but probably Euclid rather than Campanus); (ii) Witelo and Jean de Murs are particularly close to his heart – no other titles are followed by similarly laudatory comments.

What *cannot* be read out of the passage is: (i) the reasons for this respect; (ii) whether Regiomontanus saw *De numeris datis* or the *Quadripartitum numerorum* as representatives of algebra; (iii) what he thought about algebra, about its character, its legitimacy, its utility.

The Padua lecture from 1464 is of greater help. As already touched at, Regiomontanus refers to the "three most beautiful books about given numbers" which Jordanus

> had published on the basis of his *Elements of arithmetic* in ten books. Until now, however, nobody has translated from the Greek into Latin the thirteen most subtle books of Diophantos, in which the flower of the whole of arithmetic is hidden, namely the art of the thing and the *census*, which today is called algebra by an Arabic name. Here and there, the Latins have come in contact with many fragments of this most beautiful art, [...]. We also possess the *Quadripartitum numerorum*, a highly distinguished work, moreover the *Algorismus demonstratus*, and Boethius's *Arithmetic*, an introduction taken from the Greek Nicomachos. [ed. Schmeidler 1972: 46].

From the organization of the passage follows that the *De numeris datis* is understood in parallel to Diophantos's *Arithmetic*, of which Regiomontanus had just located a manuscript (letter to Bianchini, ed. [Curtze 1902: 256]). That the field dealt with by both Jordanus and Diophantos is seen as part of arithmetic (but which sublime part!) is also obvious from the text, as is the identification of *algebra* in general with the field. The *Quadripartitum numerorum* on the other hand, being grouped together with the *Algorismus demonstratus* and Boethius, is not understood as a primarily algebraic work (which would indeed be a strange characterization, algebra is only one of many topics dealt with).

It obviously suits Regiomontanus well that he is now able to refer to

Diophantos as the embodiment of algebra. The reason why is illustrated by a poem written a few years before by Peurbach, addressed to

> the nymphs [of the quadrivium] who once were sweeter to me than anything else, who taught me from the bottom, by the extraordinary ways of the Arabs, the force of the entirety of numbers so beautiful to know, what algebra computes, what Jordanus demonstrates.
>
> [ed. Größing 1983:210]

As we observe, Peurbach had noticed that normal algebra would compute (*numerare*) whereas Jordanus would demonstrate (*demonstrare*); but both ways were seen as *Arabic*. As I have shown elsewhere [1992: 11f], Regiomontanus would gladly mention Arabic contributions to astronomy (the lecture was indeed the first in a series dedicated to al-Farghānī); but in his presentation of *mathematics proper* he avoids to mention any contribution not belonging to the Greco-Latin tradition, even when he must necessarily have known better.[24] But he was sincerely interested in algebra, as can be seen in several places in the correspondence with Bianchini. The discovery of Diophantos allowed him to identify the field as an ancient discipline which only happened in his days to be "called algebra by an Arabic name".

At the same time, we observe, Regiomontanus identifies algebra as "the art of the thing and the *census*", as also done in the algebraic proofs of *De triangulis* (II.12, II.23; pp. 51, 55f). Even though Regiomontanus has read enough in the Diophantos manuscript to understand its algebraic character, his basic understanding is thus no less *al-jabr*-oriented than that of Jean de Murs. Regiomontanus was certainly a sophisticated mathematician, and even a gifted algebraist [cf. Folkerts 1980: 197–209], but he was too much engaged in his own understanding of mathematics – an underpinning for astronomy and astrology, and thus ultimately numerical and computational – to be deeply interested in the metamathematical subtleties needed for a sensitive global reading of ancient mathematics (cf. note 23). Jordanus had had to reshape the art of algebra into a "science" in order to make

[24] "Arabic" numbers and their use, for instance, are only referred to the *Algorismus demonstratus* and to Barlaam; Jābir ibn Aflah, from whom Regiomontanus borrows freely for the *De triangulis* [Lorch 1973], is mentioned with praise as an astronomer but not along with Menelaos under geometry.

it fit into *his* conception of Ancient mathematics. Regiomontanus was quite satisfied with the established orientation of algebra as a *technique for finding the number*, which was exactly what he demanded. What he needed in order to understand something as ancient was instead a *philological* or doxographic argument – that it had been dealt with by an ancient author. Jordanus's global reading of ancient mathematics had been metatheoretical – so far at least he was in agreement with the moods of his times and environment; Regiomontanus's global reading was Humanist, in agreement with *his* times and his appurtenance to the Bessarion circle.

Ramus

Humanism is also the clue to Petrus Ramus's attitude. In 1560 Ramus published an *Algebra* [Ramus 1560].[25] The book is brief (36 pages in total) and rather elementary; after introduction of the sequence of powers of the *latus* (l) until the *biquadraticubum* (l^{12}), the rest of Part I is dedicated to the presentation of schemes for the addition, subtraction, multiplication and division of polynomials; the idea and the schemes themselves though not the examples appear to be borrowed from Stifel's *Arithmetica integra* [1544: 237^v–239^r], which also seems to have supplied Ramus with other kinds of material.[26] Part II deals with first- and second-degree equations; the exposition is reasoned but based on examples; as far as the solution of the mixed second-degree problems is concerned, *Elements* II.4–6 are referred to.

In the *Algebra* itself, nothing is said about the origin of the art. From the *Scholae mathematicae* we know, however, that Ramus did all he could to make mathematics a purely Patriarchal-Greek-European enterprise.[27]

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[25] The 1560 printing is anonymous, but sufficiently close to the revised version produced by Lazarus Schoner in 1591 to confirm the authorship. At least two copies have the author's name written carefully on the title-page in ink, in a way which is intended to be mistaken for print, and which suggests systematic repair of a printer's omission (Christ Church College, Oxford, see [Ong 1958: 335, #564]; and the University Library of Copenhagen, Section II.

[26] It thus seems to be for very good reasons that Stifel is excluded from the presentation of major and minor, contemporary and near-contemporary German mathematicians in Ramus's *Scholae mathematicae* [1569: 66].

[27] See [Høyrup 1992: 15f]. On p. 117, one may notice, Ramus gives sound philological arguments that the current way to write numbers with ten signs cannot be ancient, and

Here he also mentions (p. 37)

> Diophantos, by whom we possess six Greek books, promised however by the author to be thirteen, about the admirable art of subtle and complex arithmetic that commonly is called by the Arabic name algebra; whereas from such an ancient author (he is indeed mentioned by Theon) the antiquity of the art appears.

Which consequences does this have for the way Ramus presents algebra? Few, and not very significant. Firstly, he does *not* refer to Geber, as does Stifel, or to anything else that might corroborate the Arabic origin. Secondly, he replaces the terms suspect of Arabic origin by Latin words; *radix* becomes *latus*, *census* (spelled *zensus* by Stifel and thus no longer to be recognized as Latin, and anyway a translation from the Arabic) becomes *quadratus*, *zensizensus* becomes *biquadratus*, etc. Even though the treatment is much shorter than Stifel's, in particular as far as general arguments is concerned, Ramus also takes care to conserve some of the references to *Elements* II.[28]

Instead of referring to the powers of the unknown as *cossic numbers*, the treatise starts by telling (fol. 2^r) that "algebra is a part of arithmetic that from imagined[29] continued proportions makes a certain computation of

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mentions the possibilities suggested by some that arithmetic be an invention of the Phoenicians (seemingly an ill-placed borrowing from Proclos) or the Indians. He also cites Sacrobosco for the opinion that the origin be Arabic, but adds sarcastically that the Arabs, just as they want to take possession of the whole world, may also want to take possession of all sciences.

The first three books of the *Scholae mathematicae* were published independently (as *Prooemium mathematicum*) in 1567 [Ong 1958: 362, #603]; for these as for the rest, I have used [Ramus 1569].

[28] The way Ramus uses Euclid may be derived from Stifel, but could derive through other channels from Pacioli (or some predecessor of his?). Since Thābit ibn Qurrah and Abū Kāmil, Euclidean proofs of "the cases of *al-jabr*" had referred to *Elements* II.5–6. This was also done in the *Liber abaci* [ed. Boncompagni 1857: 408], whose first proof for the case "*census* and roots equal number" was in al-Khwārizmīan style, but which then gave an alternative corresponding to Thābit's, quoting *Elements* II.5 without identifying it. In part I of the *Summa de arithmetica*, Pacioli [1523: I, 146^r] follows this arrangement of the argument, but adds the information that the first demonstration agrees with *Elements* II.4 (misprinted at least in the 1523 edition as I.4). Stifel (fol. 244^v) points to *Elements* II.3 while noticing that II.4 might serve just as well. Ramus refers to II.4 only (fol. 14^r).

[29] The Latin word is *figuratus*. Ong [1958: 166, 334] interprets as "figurate numbers" (triangular, square, pentagonal, cube numbers etc.), and sees a dilemma because this evidently does not fit; my alternative translation agrees with the situation of the word as an epithet to *proportion*,

its own”; thus the reference to the classical concept of a continued proportion, already present but secondary with Stifel, becomes primary.

In a way, this strategy is similar to what we found with Regiomontanus: even Ramus was too busy with his own kind of mathematics to make a serious global reading of ancient mathematics. There is, however, and important difference. Regiomontanus was able to find *algebra* in both Jordanus and Diophantos; nothing suggests that Ramus had had the occasion to do more than read *about* Diophantos.[30] Algebra in its *al-jabr* form was even more adequate for Ramus (given his infatuation with pseudo-utility and his lack of interest in more than heuristic proof) than it had been to Regiomontanus; if we look for its appearance *in actu* and not as a mere name in the *Scholae mathematicae* we shall find it in two places: p. 143, where algebra is told to be a more convenient tool for solving a Greek arithmetical riddle than proportions; and in books XXIV and XXV (pp. 274–283), where it is used to explain *Elements* X (another probable borrowing from Stifel).

In general, and not only when algebra is concerned, Ramus refers just as much to ancient letters in general as to mathematical works; we may claim that whatever global reading he attempted was not a global reading *of mathematics* but of antiquity as a whole – in good agreement with his universalist programme of subsuming all knowledge under a reformed version of rhetoric.[31]

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not *number*, as well as with seventeenth-century readings. Thus Alsted [1620: 741], when paraphrasing the passage in his first encyclopedia, speaks of “numbers of figurate *value*”. When the same idea is repeated in his second encyclopedia [1630: 844], it even induces him to revise the concept of figurate number, defining it (p. 828) as a product, whose “factors are called its sides or roots”, in agreement with the definition given by Lazarus Schoner [1599: 139] in the *De numeris figuratis liber*. This work was written as a kind of complement or underpinning to Schoner’s revised edition of Ramus’s *Algebra* and *Geometry* and published in one volume with these – a volume of which other traces can also be found in the 1630 encyclopedia.

[30] I have not been able to trace from where Ramus has got his information; the detail that only six books are extant is not found in Regiomontanus’s Padua lecture as published by Petreius in 1537.

[31] Though put in other words, this interpretation of Ramus’s relation to mathematics was already suggested in stronger (unduly strong) form by Neal Gilbert [1960: 85f]. In order to defend Ramus’s *Elements* with explanatory examples instead of proofs he observes that “the

Cardano

Without any pretence of completeness, the introductory passages of two other works from the central decades of the sixteenth century and their relation to the style of what follows may illustrate the far from convergent impact of Humanism on the conception of algebra – in chronological order, Cardano's *Ars magna* from 1545 and Nunez's *Libro de algebra* from 1567.

The introductory passage of the *Ars magna* runs as follows:[32]

> This art originated with Mahomet the son of Moses the Arab. Leonardo of Pisa is a trustworthy source for this statement. There remain, moreover, four propositions of his with their demonstrations, which we will ascribe to him in their proper places. After a long time, three derivative propositions were added to these. They are of uncertain authorship, though they were placed with the principal ones by Luca Pacioli. I have also seen another three, likewise derived from the first, which were discovered by some unknown person. Notwithstanding the latter are much less well known than the others, they are really more useful, since they ⟨teach the solution of cubes, numbers, and squares of cubes. In our own days, Scipione del Ferro of Bologna has found the chapter on cube and things equal to number, a truly beautiful and admirable thing⟩ [...].

No doubt Cardano has learned from Humanism (here and elsewhere); but when he has to choose between Humanist method and Humanist ideology, he opts for solid and critical philology. He has little use for Diophantos's sophistication buried in problem solutions (assuming that he really had

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proofs in the *Elements*, in Ramus' days, were usually considered to be scholia, or parts of a commentary, upon the text, written by a certain Theon [...] who had commented upon Euclid in this rather laborious fashion. [...] When Peter Ramus omitted the proofs from his edition of Euclid, then, he was simply dropping, so he thought, an unnecessary commentary with which the student need not be burdened!" This insinuation that Ramus had only a (mistaken) philologist's view on *Euclid* and was not able to see that the whole structure of the work presupposes proofs is mistaken; in book I of the *Scholae mathematicae* (p. 39) Ramus states that Proclos, in his commentary, has Euclid's original proofs, and that the traces of these can be found in the Theonian proofs. No doubt thus in Ramus's mind that Euclid made proofs; when discarding them Ramus did not believe to follow the ancient ways but to correct Euclid's "Platonic error".

[32] I quote Witmer's translation [1968: 7f], correcting a passage where I find it unduly free (in ⟨⟩); my correction is based on the second edition from 1570, reproduced in [Cardano 1663: 222].

access to his mathematics, which may be doubted[33]); in any case, he abstains from irrelevant namesdropping, and traces the real development of the discipline he is about to renew as far as it is known to him (he had inspected the manuscript of the *Liber abaci* used by Pacioli, and also Pacioli's own printed book, but apparently not the various fourteenth- and fifteenth-century *abbaco*-manuscripts).

Even if he had not had to choose, it can be added, Cardano might not have agreed with Humanist ideology; in the *De subtilitate* from 1550 he affirms that "every truth is divine" [Cardano 1663a: 607]; already in 1535 he had transgressed the prevailing rules of conduct in the *Encomium geometriae*, honouring (under *this* heading!) not only al-Khwārizmī but also Fibonacci and Pacioli (without the usual denunciation of his language) [1663: 443f].

Cardano thus ascribes no ancient ancestry to algebra; the influence of ancient mathematics, instead, is found in the style and the contents. Comparison of Cardano's geometric proof for the case "square and things equal to number" [1663: 229] with counterparts in other treatises shows him to be much more precise in the geometrical argument. The usual completed gnomon is investigated with reference to def. 1 of *Elements* II and to prop. I.43 (the gnomon theorem), after which II.4 is used. Similar observations could be made on the other cases (even more radically for the case "things equal number plus square"); without having any ideological axe to grind, Cardano thus shows us to be familiar with the norms of ancient geometry and to follow them as far as the topic allows.

Nunez

In the dedicatory letter of the *Libro de Algebra en Arithmetica y Geometria*, addressed to the "muito alto e muito excellente Principe o Cardeal Iffante Dom Anrique", Nunez [1567: a ii] states that

> among all the books I have composed in the mathematical sciences, most high and most excellent prince, none is as useful as the present one on algebra, which is an easy and concise computation allowing to find an unknown quantity in any arithmetical and geometrical problem, and in every other art

[33] Diophantos seems not to be mentioned, neither in the *Ars magna* nor in the Ferrari-Tartaglia *Cartelli* [ed. Masotti 1974].

> that uses computation and measurement, such as cosmography, astrology, architecture, and commerce. And since the principles of this sublime art are drawn from Euclid's books on the *Elements*, we may put them to use in these same books, and in those of Archimedes. Algebra is an Arabic name meaning restoration, by which, through subtraction of the excess, and restoration of what has been taken away, we come to know what we search for. Others suppose that it is called thus because it was invented by a Moorish mathematician whose name was Geber, and in some libraries they have a small treatise in Arabic containing the topics with which we deal. But Johannes Regiomontanus, in the lecture he made in praise of mathematics, mentions two books written by Diophantos, a Greek author of this art, which have not yet been published. The first book that was printed on algebra is the one Fra Luca de Borgo [Pacioli] composed in Venetian language, but so obscurely and to such an extent without method that today, more than 60 years after its printing, very few in Spain have knowledge of algebra.

That Nunez's reference to Diophantos is based on rumour is made quite clear. Whether he inserts it because he himself wants algebra to be ancient is far from evident, however; he may well have accepted Regiomontanus's claim on sheer authority. So much appears from the following text, however, that whatever intentions he had did not influence his way to deal with algebra very much. We may use again the geometrical demonstration for the case "*census* and things equal number" (pp. 6–8) as our touchstone. The diagram – the usual completed gnomon – is explained with no more geometrical precision than by Pacioli (but more clearly). Two differences can be taken note of. Firstly, Pacioli as well as his predecessors had argued on a particular example ("10 things"); Nunez's argument is general. Secondly, Nunez eliminates the alternative proof based on *Elements* II.5; in return he tells that the same is shown regarding numbers by Campanus in V.16[34] (commenting however that this cannot be used for numbers consisting of indivisible units if the number of "things" is odd).

The elimination of the reference to *Elements* II.5 and the reference to an acknowledged scholastic insertion do not support any assumption that Nunez was very eager to legitimize algebra by connecting the discipline

[34] This is one of the propositions which Campanus took over from Jordanus's *De elementis arithmetice artis* [*Euclidis Megarensis ... Elementorum libri xv*: 230]. Nunez obviously knows that the proposition is an insertion, probably because he has used one of the editions containing the Campanus and the Zamberti versions in parallel.

to antiquity; elsewhere his argument for legitimacy relies indeed on the *efficiency* of the algebraic technique.[35] The tendency to replace specific with general formulations (even more visible when we come to the use of algebra in geometry, pp. 227 onwards, where the format becomes that of the *Data* – "If ... be known, then ... will also be known") probably reflects Nunez's general notion of mathematics, just as the Diophantos reference can be supposed to reflect his level of Humanist culture. Nunez appears to be another instance of the mathematician who is more interested in making his own reform of mathematics than in questions of legitimacy; the global reading of ancient mathematics ("space *S*") only influences him in so far as it has been accepted by and absorbed into that contemporary mathematical culture to which he relates ("space *T*").

VII. Towards modern algebra

In Christian Wolff's *Mathematisches Lexicon* [1716: 35–37], algebra is dealt with under two headings: "*Algebra numerosa*, common or old algebra, or algebra in numbers"; and "*Algebra speciosa*, the newer algebra". The former comes from the Arabs, and ends with Pacioli and Stifel; the second is told to take its beginning with Viète, and to have been put in better shape by Harriot and Descartes. Cardano and Nunez (none of whom are mentioned) become transition figures, whom I have chosen to group with the old numerical algebra. They were certainly searching for new ways, but still within the old framework; even Cardano's work on third- and fourth-degree problems is a continuation, not only in del Ferro's and Tartaglia's footsteps but of an interest in higher-degree problems manifested since the fourteenth century [Franci & Toti Rigatelli 1985, *passim*; Franci 1985]. Forthcoming renewal was rather adumbrated by the metamathematical aspects of their works as discussed above.

[35] Thus also though vaguely in the passage quoted from the dedication; this passage *does* argue from the Euclidean basis of the art, but with the specific point that what *comes* from Euclid may legitimately be *applied to* Euclid and Archimedes, i.e., to geometry. But the motive for doing so is the efficiency of the algebraic technique even in this field.

Bombelli

Also arbitrary is the assignment of Bombelli's *L'algebra* [1579] to the "new algebra". Even Bombelli is moving within the traditional framework – but in a way that demolishes precisely its function as a framework, which may justify the decision to categorize him as "new".

Bombelli's systematization and extension of earlier work on third- and fourth-degree problems is well known; but on that account what he does is to bring the framework to completion, in continuation of Cardano – *non tollit sed perficit*, in St Thomas's phrase. In a different metaphor, this aspect of the work belongs on the level of *tactics* – adequate application of available means.

Strategic (concerned with the planning and procurement of means for future action) is instead his introduction of a new formalism for the powers. Since the fourteenth century, names at least for the powers 1 to 6 had been in use, though still inconsistently in the later fifteenth century.[36] Pacioli [1523: I, 143^{r}] had extended the system and made it consistent (but not very manageable in the rare cases where powers beyond the sixth would occur); this system was borrowed with corrections by Stifel and Ramus. It was customary to observe that the system could be extended *ad infinitum*, but the terminology was evidently inadequate for that purpose. Bombelli borrows Pacioli's system (changing *cosa* into *tanto* and *census* into *potenza*), but at the same time he undertakes the radical step to arithmetize the whole topic, introducing the notation $\underset{\smile}{n}$ for the *n*th power. Even though the following generations were to change the notation in order to accommodate the presence of several variables, the principle that powers were counted by numbers and not designated by individual names was conserved.[37]

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[36] One may compare the terminology of Piero della Francesca [ed. Arrighi 1970: 84f] with those of Benedetto da Firenze and Regiomontanus. Piero uses *censo di cubo* for the fifth power and *cubo di censo* for the sixth; with Benedetto, *cubo di censo* designates the fifth power, and *cubo di chubo* the sixth [Franci & Toti Rigatelli 1983: 301]. With Regiomontanus [ed. Curtze 1902: 280], *census de cubo* designates the fifth and *cubus de cubo* the sixth power.

[37] Bombelli, it is true, is not the first to arithmetize the notation for powers – it is already done by Chuquet in the *Triparty* from 1484 [ed. Marre 1880: 737], who points to the disadvantages of the traditional notation by names or specific symbols. But whereas Chuquet's influence seems to have faded out quickly, Bombelli was read.

The geometric proofs for the mixed second-degree cases presents us with a different kind of innovation. “Even though this science is arithmetical (as it is called by the Greek author Diophantos, and by the Indians[38]), it does not follow that the whole thing cannot be proved geometrically (as does Euclid in the second, the sixth, and the tenth)” (p. 241) – and in order not to dissatisfy the reader, Bombelli undertakes to show how. As Cardano, he gives the reference to *Elements* I.43 and other relevant propositions. But he goes further, and along with the traditional geometric demonstrations he shows how the solutions can be *constructed* geometrically.

Finally, as is also well known, Bombelli replaces the usual names-dropping with real *use* of Diophantos, even replacing all the practical or pseudo-practical questions that his first manuscript for Book III had contained with problems borrowed from Diophantos [Jayawardene 1973]. When discarding this veil of

> human action and business (like selling, buying, barter; exchange; interests; defalcation; alloys of money and metals; weights; partnership, both with loss and with profit, games, [...]),

Bombelli [1579: 414] explains to

> have had in mind to teach truly the discipline of the major part of arithmetic (called ‘algebra’) in imitation of the ancient authors, and a few of the moderns; because the others, acting as told above, have been practical rather than scientific; and today it is seen in every discipline that theory is taught, and not practice, from the supposition that the human intellect should be like that; that it should come on its own (when in possession of the theory) to the usage of practice; and particularly in the mathematical disciplines this should be believed, since (as is well known) they lean toward theorizing.

Neither the arithmetization of the calculus of powers nor the more extensive use of theoretical geometry is necessarily to be explained from Bombelli’s personal attitude toward ancient mathematics; the former is a parallel to Nunez’s preference for the general, the second is already adumbrated in the *Ars magna*, and in both cases contemporary notions

[38] This puzzling reference to the Indians is also found in the preface. As Bachet [1621: a iiij^{r}] points out, Bombelli has mistaken “affected and silly” Byzantine scholia with reference to the multiplication with Hindu numerals for Diophantine notes.

about what mathematics should be appear to be in play,[39] rather than independent global readings of ancient mathematics. The abstraction of book III, however, is an indubitable consequence of Bombelli's *direct* contact with antiquity (*viz.* with Diophantos). Without this encounter and the impression it made on him, he would neither have chosen *theory* instead of practice as the aim of the discipline nor have opened it toward implicit theory of numbers, as he did by the inclusion of numerous Diophantine indeterminate problems.

In contrast to what we have seen with Jordanus, however, Bombelli's was no sensitive global reading of ancient mathematics as a whole. What impressed him was a local reading of a particular mathematician; whatever global ancient influence can be found in his work is probably indirect, going via "space *T*". We may even observe that he does not accept the consequence of that orientation toward pure theory which he finds in Diophantos *and embraces*. The "Platonic error" (to speak with Ramus), i.e., the preference of theory for practice, was so far beyond the horizon of this early Modern mind that he can only defend theory as the best tool for practice – "es gibt nichts praktischeres als eine Theorie". None the less Bombelli was probably the first algebraic author since Jordanus to transform the topic in a way that was marked in depth by his direct reading of ancient mathematics.

Viète

Bombelli had thus undermined the traditional framework without being fully aware of it; prepared the tools for attacking problems not yet imagined; and opened for theoretical developments in which he only engaged himself to the extent he was forced to by following Diophantos. It is thus for good reasons that neither Wolff nor later historians take *him* as the architect of modern algebra. Since Wolff, it has been customary to ascribe this role to Viète, im whom aims, actual creation and impact agree to a much larger extent.

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[39] Notions and norms which had certainly been influenced by the style of ancient mathematics as read by generations of mathematicians; but which had *themselves* become institutionalized. Centuries of sustained efforts to understand "space *S*" had transformed the character of "space *T*" itself, in agreement with the Heidegger-Gadamer version of the Hermeneutic circle.

Viète's conception of his own accomplishment is formulated in the dedicatory letter of the *In artem analyticen isagoge*. As it is expressed there, existing algebra was "so defiled and polluted by barbarians" that he found it necessary "to bring it into a completely new form". The enterprise was necessary, because, as all mathematicians knew, "under their *Algebrâ* or *Almucabalâ*, which they extol and call the great art, incomparable gold is concealed, which however they cannot find at all" [ed. Hofmann 1970: XI].

Diophantos could not serve here without being recast. As stated in chapter V (p. 10),

> Diophantos exercised zetetics most subtly in those books that are contained in the *Arithmetic*. There, it is true, he exhibits it as if in numbers and not in species (which none the less he used), so that his ingenuity and quickness of mind should be more admired: for things that appear very subtle and abstruse in numerical logistics are quite everyday and often obvious in specious logistics.

This is certainly meant as blame and as an argument that Diophantos is not to be imitated but rather to be exposed and robbed of his secrets. Instead, Viète falls back for the overall shape of his "new art" on more fundamental ideas borrowed from ancient mathematics and philosophy: the concepts of *species* and *analysis*.

What is meant by "species" has been much discussed [see Witmer 1983: 13 n.8], but the term is indeed explained by Viète in the beginning of chapter IV (p. 4): "Numerical logistics is that which is presented by means of numbers, specious logistics that which is presented by *species* or the forms of things, possibly by means of the elements of the alphabet". As in the doctrine of the "multiplication of species", the *species* is a pure form, which can be filled out by any number. Through this artifice, Viète provides a philosophical legitimation for the use of letter symbols that comes close to the "place-holder" of modern mathematics education and is much more satisfactory than the alternative "imagined" number used by Ramus and other near-contemporaries. "Specious logistics" is thus not "symbolic algebra" in the sense which contrasts it to "rhetorical" and "syncopated" algebra and which excludes Jordanus's *De numeris datis*; it is a category to which both this work and Viète's own version of algebra belongs.

Analysis is told in chapter I (p. 1) to be a method invented perhaps by Plato and given its name by Theon, "the assumption of what is searched for as if it were given, and then from the consequences of this to arrive at the truly given". We need not go into the details or into the discussion of whether Viète's concept coincides with the ancient method; what is important for our present purpose is that he immediately adds a third type of analysis ("rhetic and exegetic") to the two he finds with the ancients ("zetetic" and "poristic).

The "species" idea is convenient, but it is certainly a subterfuge, used to dress up a letter symbolism the necessity for which nobody would be able to derive from the Aristotelian concept – since the Aristotelian species is unique, how can the species of number occur in two or more mutually independent copies within the same expression, as *a*, *b*, etc.? The recourse to analysis is not quite as obviously an *a posteriori* invention – familiarity with the ancient metamathematical discussions may have provided some inspiration. But the rapidity with which the concept is transmuted suggests that its role cannot have been very important. Viète's way to make his new art agree with ancient standards seems to have influenced neither the approach nor the mathematical substance to any substantial degree. His newfangled Greek terminology, though less glaringly do-it-yourself than what would be found with an Athanasius Kircher in the mid-seventeenth century, already looks like a Baroque external adornment.[40] Viète was, like Jordanus, not only an excellent mathematician but also one for whom metamathematical reflection played a central role; but his metamathematics seems to have developed from reflection on contemporary mathematics. It was certainly dependent on ancient norms, but mostly through the way these had already become self-evident within a particular mathematical culture; his ingenious, newly invented references to antiquity may have served their role as legitimization – but hardly much more.

[40] Viète's fascination with pseudo-Greek neologisms may be compared with Agricola's excuses in the preface to the *De re metallica* [trans. Hoover & Hoover 1950: xxxi], dated as late as 1550, for introducing the neologisms needed if mining processes are to be described – neologisms as innocent as regularly formed *nomina agentis* of Latin verbs. The contrast leaves no doubt that Viète's habit points toward the following, not the preceding epoch

Descartes

After 1600, algebra in the shape of *algebra speciosa* was not felt by those who participated in its development to be in need of further ancient legitimization. When redefining in his *Geometrie*[41] "squares" and "cubes" as line segments, Descartes tells that he preserves these names as "usités en l'Algèbre"; when discussing in the beginning of Book II the class of curves that can be legitimately used in geometry, moreover, he censures the ancients for having rejected as "mechanical" all curves beyond the line and the circle, probably, thus Descartes' explanation (p. 317), because they first considered the (Archimedean) spiral, the quadratrix and similar (transcendental) curves and then did not notice the distinct character of those algebraic curves of which he himself is going to make use in the following. Clearly, he makes no effort to borrow for his new use of algebra the glory of the ancients, whose lack of method was the cause of "beaucoup d'obscurité, et d'embaras" in their writings (p. 306). No doubt that the ultimate root of the distinction between legitimate and illegitimate curves is to be found in ancient mathematics; but its acceptance by Descartes is an evident "space-*T*" effect.

VIII. Baroque aftermath

As far as the development towards modern algebra is concerned, this is thus the end of the story; from Descartes onward, algebra is too well established in one or the other form to need external legitimization, and thus no longer that mirror through which we have so far examined how proto-Modern and early Modern mathematicians read ancient mathematics globally. Mostly, as we have seen, they would read it in ways that fitted the kind of mathematics they were doing anyhow and through those norms (etc.) for mathematical works which ruled that particular discursive space ("*T*") and that particular community within which they moved.[42] Ancient

[41] P. 299 in the 1637 edition, which I cite from the facsimile in [Smith & Latham (eds) 1954].

[42] At the risk of being pedantic I shall emphasize the following points:

(i) It was at least as meaningless in the late Middle Ages and the Renaissance to speak of *one* mathematical community – *maestri d'abbaco* and university-trained astronomers, to take mathematics certainly influenced their mathematics and their ideas about mathematics, but mostly in the form in which it had been accepted within this space and this community.[43] In only two cases – *viz* Jordanus and Bombelli – would the direct reading of the ancients have an impact on the kind of algebra that was done; and only in Jordanus's case is it possible to maintain that the influence came from the global reading.

We might therefore close the tale at this point. However,

> Hegel remarks somewhere that all the great events and characters of world history occur, so to speak, twice. He forgot to add: the first time as tragedy, the second time as farce.[44]

As readers of Shakespeare will know, farce is not necessarily less valuable than tragedy; quite apart from the merits it possesses on its own it may

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one example, had few norms, tasks, habits and traditions in common.

(ii) The communities which *can* be traced are only to be understood as *mathematical* communities in a very loose sense – even if we forget about Regiomontanus's roots in university astronomy and court astrology, the Bessarion circle was primarily a particular Humanist environment though open to Regiomontanus's mathematical interests.

(iii) Individual mathematical workers would, then even more than now, be likely to be members of intersecting communities, among other things because social roles did not agree with epistemologically defined borders. Regiomontanus (to cite the same example) knew perfectly how to distinguish mathematics from astronomy and astronomy from astrology, but he would engage (not only socially but also emotionally) in all fields.

(iv) "Communities" are constituted by people as discursive spaces are constituted by actual pieces of discourse. That does not reduce them to *arbitrary* sociologists' shorthands; but shorthands they are, and though useful conceptual tools they must be handled with increasing theoretical care as the number of constituents grows smaller (as does the membership in all "scientific" communities when we go backwards in time). At the extreme limit, a discursive space may be the construction of a single individual on the basis of select readings (Jordanus seems to come close to this extreme).

[43] Since it was not directly visible in the sources that were discussed above, one particular influence has so far gone unmentioned: mathematical writers might provide their mathematics with ancient apparel not just in order to make it agree with their own norms but also to make it agree with (what they expected to be) the norms of a community from which they wanted acceptance. There is no reason to doubt the sincerity of Luca Pacioli when he adds Euclidean references to his reproductions of Fibonacci or publishes the *Elements* – he insists on doing so much else showing that he had been brought up in the practical tradition; but he may *also* have done it in order to agree with the expectations of that Humanist and courtly environment within which he wanted acceptance.

[44] The initial passage from Marx's *Achtzehnte Brumaire des Louis Bonaparte*, as translated by Ben Fowkes [Fernbach (ed.) 1973: 146].

also throw new light on the action of the tragedy. It may therefore be worthwhile to have a look at the way algebra is dealt with in Caramuel's *Mathesis biceps* [1670], a work of well beyond 1800 folio pages.

Caramuel was born in Madrid in 1606 and died as Bishop of Campagna and Vigevano in 1682. He was a prolific writer on many subjects, much in the vein of Athanasius Kircher, and like Kircher also a theoretician of Baroque poetry. In one of his writings on this theme he formulates the programme that "the Machine of the World overflows with Proteus; let us therefore seize a Proteic pen in order to sing the praise of Proteus".[45] He might not have been offended by Galileo's famous gibe against Horatio Grassi, namely that Grassi saw philosophy as "a book [...] like the *Iliad* and *Orlando furioso*, in which the least important thing is whether what is written there be true".[46] These quotations are useful for understanding Caramuel's treatment of algebra.

The *Mathesis biceps* falls in two volumes, of which the former deals with *mathesis vetus* and the second with *mathesis nova*. Algebra is dealt with on pp. 97–206 under the heading of "old mathematics", which already indicates that *algebra numerosa* is meant. This is noteworthy in itself, since other parts of the work betray familiarity with astronomers and mathematicians of the mid-seventeenth century; Caramuel may not have been competent to follow the higher algebra of Descartes' *Geometrie*, but that he should have been unaware of the existence of a new kind of algebra as part of the *new* mathematics of the epoch is highly unlikely.

But what Caramuel does is amazing even from the perspective of *algebra numerosa*. His 108 folio pages on the topic never get beyond the first degree in its actual subject-matter;[47] this cannot be because he is unable to

[45] *Primus Calamus ob oculos ponens Metametricam* [Caramuel 1663], "Apollo analexicus" p. 1; cf. [Koch 1994: 90].

[46] *Il saggiatore* [ed. Favaro 1890: VI, 232].

[47] There is, however, a list of the names for the powers unknown (until the ninth power), apparently derived from Ramus's names but emended; moreover, the usual cossic symbols are listed, together with two alternatives. The first possibility is to write *a* for the first power, *aa* for the second, *aaa* for the third, etc. This had been proposed by Johannes Geysius in his *Cossae libri iii. De fictis numeris arguentibus veros*, a short treatise inserted in Alsted's second encyclopedia [Alsted 1630: 865–874]; Caramuel points out that this is easily misread, and suggests instead ´, ˝, ˝´, $^{\prime\mathrm{v}}$, $^{\mathrm{v}}$, $^{\mathrm{v}\prime}$ (overlooking that Geysius uses his system to work with several

understand traditional second-degree *al-jabr* algebra (some of his chapters show him to be a fairly competent mathematician, though with interests which do not square too well with those of his contemporaries) or had no sources for it.[48] Caramuel even makes use of the familiar geometrical diagram *à la Elements* II.4 and al-Khwārizmī (p. 130), not however for solving an equation but when showing how to extract the square root of a second-degree polynomial.

In agreement with his Proteic predilections, Caramuel feels free to choose, among several possibilities, what *he* finds algebra should be; in another chapter where a plurality of options is present he states (p. 39) that

> I might propose many: but three please me, which I should especially exhibit and elucidate. Others may consider others, and even we, if occasion will allow it, will think about them and explain them.

Even when his choice appears to be compulsory – namely when he is to decide between the Ptolemaic, the Tychonian and the Copernican world system – he presents his verdict (p. 1440) as an instance of personal predilection and Baroque subjectivity:

> I am not the one who wants that which has been censured by the Church. The Copernican system will hence be repudiated, and the two others remain under judgment. The Ptolemaic system is improbable, since nobody can deny that Venus and Mercury are moved around the Sun. The Tychonic system thus remains.

When interpreting Caramuel's attitude we should not mistake subjective choice for non-commitment to truth; when taking over Ramus's notion of the "imagined continued proportion" as the core concept of algebra he changes it into "abstract proportion" precisely because he cannot accept that true conclusions be derived as the (necessary) consequences of false assumptions (pp. 99, 109f, and *passim*). This problem becomes urgent, not so much because of the "falseness" implied in Ramus's explanation (as

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variables). He also brings the usual schemes for computation with polynomials, including the finding of their square and cube roots.

[48] In general it would be next to impossible to find a presentation of algebra not including at least second-degree equations; moreover, Geysius treats both second- and third-degree problems (the latter inconsistently, in pre-Cardano manner).

repeated in slightly varying form by Alsted and Geysius, both cited by Caramuel) as because of his chosen understanding of algebra, as *based on the rule of false position* (p. 99).[49]

So far we have seen two farcical imitations of the development of modern algebra: Viète's "species" or abstract form being transformed into "abstraction", neither better nor worse in principle; and Bombelli's arithmetization of the powers, transformed into the use of *apices* extended like the Roman numerals – less elegant than Bombelli's notation, but identical with the system used by Caramuel himself (p. 61) and by numerous other mathematical and astronomical authors since 1571 for the subdivisions of the degree and accepted since then by everybody [Cajori 1928: §§513f]. The style and "action" of the "farce" are thus in themselves no less noble than those of the "tragedy"; what makes the whole thing farcical is that the noble action takes place among shopkeepers and servants – that is, metaphors apart, that the innovations are applied to a mathematical substance which had done without them for centuries and did not seem to ask for them. I technical terms, what goes on is a travesty, "a village girl in the dress of a princess", the genre which Perrault named a "farce of the second kind".[50]

The same kind of farce occurs when Caramuel establishes the ancient origin of algebra. He does so in two ways, one explicit and one implicit.

The explicit argument is a travesty of Regiomontanus's reference to Diophantos Diophantos clearly could not serve an algebra identified with the rule of false; instead Geysius is quoted to the effect that Greek epigrams demonstrate "the fondness of antiquity of cossic arithmetic".[51] In a way

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[49] I have not been able to find any earlier occurrences of this singular definition, which of course corresponds to Caramuel's restriction to the first degree. Alsted (whom Caramuel chides on p. 120 for being so ignorant of the topic that he has to ask Geysius to treat of the matter in his *Encyclopaedia*) refers [1630: 844] to algebra as "a certain special rule of three"; but Caramuel presents not only the single false position but also in detail the double position. His definition appears to have resulted from a fully idiosyncratic choice.

[50] "Burlesque de la seconde espece, où le sujet qui est bas et rampant se traite d'une maniere sublime et relevée" [Perrault 1688: III, 301f] – as when Homer describes in heroic verse the combat between Odysseus dressed in rags and the villain Iros.

[51] [Alsted 1630: 874]. Ironically, the epigram quoted as evidence by Geysius, "῾Ημίονος κὴ ὀνος φορέουσαιο ἰνον ...", is the only epigram from [Bachet de Méziriac 1621] which is not

the argument is better than Regiomontanus's reference to Diophantos, since Regiomontanus's notion of algebra referred to the *al-jabr*, not the Diophantine type: the Greek arithmetical epigrams, on the contrary, appear to fit precisely the kind of "algebra" Caramuel has chosen to present.

The implicit argument has to do with the problem inherent in this "appear". The epigrams, indeed, only state the riddle and tell no method. The certified ancestry of the *regula falsa* was thus not Greek but (as far as Caramuel could know) to be found within the *abbaco* and cossist tradition. This is also what Caramuel suggests when he introduces the rule (pp. 109–116): the dress of the problems used to exemplify this method (rather, these methods) is exactly of the type familiar from the *trattati d'abbaco*. However, this explanation precedes the metatheoretical introduction of algebra proper (as understood by Caramuel), including the discussion of powers and symbols for these as well as the methods for computation with polynomials. Then follows another, much more extensive array of problems (pp. 134–176), to be understood as the questions representing "algebra" or "abstract proportionality". Their mathematical structure is no different from the problems representing the *regula falsa*.[52] Their dress, however, is ancient. Gone are the references to merchants unless they are from Athens and Thebes. Instead we get "Theseus's sea-voyage", "Hiero's crown", "Alexander's age", "Homer's travel", etc. Without being asserted explicitly – the implicit argument is of course much too subtle and allusive to allow that – it is thus exhibited *ad oculos* that (Caramuel's version of) algebra can be regarded as ancient, or at least that it fits legitimately into a scholarly culture based on antiquity. Strict historiography will certainly deny that any positive link to antiquity is established; but in view of the generally ambiguous nature of the relationship of Baroque culture to antiquity we have to accept that Caramuel manages to insert *algebra as*

found in Codex palatinus gr. 391, and thus the only one of whose ancient origin we are not certain [cf. Tannery 1893: II, x] (Bachet borrowed it from Planudes's anthology, printed in Florence in 1494 and excerpted in numerous sixteeenth-century *florilegia*). Though not identical, Geysius's Greek spelling comes so close to Bachet's that he may conceivably have used this source; his Latin translation, however, is different. The same epigram, in strongly deviant spelling and with yet another Latin translation, is found in [Ramus 1569: 143]

[52] With one noteworthy exception: two problems (pp. 144–146) are of the type "leo in puteo", and thus *less* fit for simple algebraic treatment than the ones representing the rule of the false.

understood by Caramuel into *antiquity as understood by the Baroque*.

The argument, no doubt, has nothing to do with mathematical substance nor with any (global or local) reading *of mathematics*. It is not even philological, as Regiomontanus's reference to Diophantos and Geysius's and Caramuel's to the arithmetical epigrams. If we are to classify it, we may speak of pseudo-philological allusions. Yet even this corresponds, as travesty to tragedy (in the present case perhaps to unwillingly farcical tragedy?), to something we have encountered above: Viète's pseudo-Greek neologisms, already characterized as proto-Baroque.

Regiomontanus's and Viète's (etc.) attempts to relate to antiquity go together with indubitable expansion in mathematical performance. We therefore tend to accept the ancient inspiration as a naturally inherent aspect of early Modern mathematical progress, perhaps even as a moving force. Caramuel's travesty, by establishing quite analogous links but within a framework that does not involve (nor *tries* to involve) a progression of mathematical knowledge, will serve to remind us that such links may often have been quite as coincidental to progress as is Aristotle's lightening to the walking of the man. Just as the imminence of a thunderstorm may make us shorten our walk, the obsession with ancient legitimacy may even have been a burden and an impediment in many cases. That Jordanus and Bombelli were pushed (each in his own way) toward their actual projects by this obsession is indubitable; but they should not be taken as instances of a general pattern. Algebra, the "new" (i.e., "non-ancient") mathematical science *par excellence* of the incipient Modern epoch, remained new.

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