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Jacopo de Florentia, *Tractatus algorismi* (1307), the chapter on algebra

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To Ahmed,
Raffaella,
and Laura

Contents

Introduction

The following is an edition and English translation with a minimal mathematical and historical commentary of what is the earliest European vernacular algebra so far known.[1] It comes from the manuscript Vat. Lat. 4826, which contains a copy of Jacopo da Firenze's *Tractatus algorismi*. The treatise as a whole was described by Louis Karpinski [1929].

The incipit (normalized as the edition below) reads as follows:

> Incipit tractatus algorismi, huius autem artis novem sunt speties, silicet, numeratio, addictio, subtractio, ⟨mediatio,⟩[2] duplatio, multiplicatio, divixio, progrexio, et radicum extractio. Conpilatus a magistro Iacobo de Florentia apud Montem Phesulanum, anno domini M^{o}CCC^{o} VII^{o} in mensis septenbris.

In translation:

> Begins the treatise on algorism, which art consists of nine species, namely, numeration, addition, subtraction, ⟨mediation,⟩ duplation, multiplication, division, progression, and root extraction. Compiled by Master Jacopo da Firenze at Montpellier,[3] in the year 1307 in the month of September.

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[1] It is a pleasant duty to express my gratitude to Warren Van Egmond for comparing my transcription with a photocopy of the manuscript and correcting a number of errors.

[2] Inserted in agreement with the Florence manuscript, and needed in order to fill out the number of nine species.

[3] This has been the usual reading of what appears as *Montem Phesulanum* in the Vatican ms and as *Montem Pesulanum* in the Florence ms; but as pointed out by Enrico Giusti (oral communication), the former spelling might rather suggest Fiesole.

A problem about travelling between Rome and Montpellier (fol. 24^{r-v}) is of course no proof that Montpellier is really meant, only evidence that Rome and Montpellier, together with Florence, Bologna, Avignon, Toulouse, "overseas", Genova, Aigues-Mortes, and Lucca, constitute the horizon of travelling of the problems. Florence and Montpellier are the only two places that turn up twice; other locations, from Nîmes to Sicily, are only mentioned as domiciles of their currency and measures. So, irrespective of whether Jacopo was actually writing in Fiesole or Montpellier, the commercial environment that constituted his base was certainly centred on Languedoc.

The rest of the treatise is in Italian.[4] The manuscript is written in a single hand, conscientiously copied around 1450.[5]

Two other manuscripts of the treatise are known: Ricc. 2236 of the Biblioteca Riccardiana (Florence), and Trivulziana 90 (Milan). An edition of the former was made by Annalisa Simi [1995]; the latter is described in [Van Egmond 1980: 166*f*].[6]

What makes the Vatican manuscript interesting is that the Florence as well as the Milan manuscript jump directly from mensuration to alligation; in the Vatican manuscript a section on algebra is inserted between the two.[7] After the alligations, the Vatican manuscript contains another collection of mixed non-algebraic problems and rules (fols. 50^v–58^r) which are not found in the Florence-Milan version. Two of these are rendered in the appendix to the present edition.

Evidently, this suggests the possibility that the algebra and the latter

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4 The repeated confusion between "proportion" and "proposition" suggests that the author was not too familiar with scholarly Latin.

5 At times the copyist has corrected his first spelling – seemingly when he has used the habitual spelling of his original and discovers that the original deviates from its own norms (cf. also note 20; a few dittographies have gone unnoticed, it is true). Dating according to Warren Van Egmond [1980: 224], based on watermarks.

6 After the manuscript was finished I heard from Jean Cassinet (personal communication) that he is preparing a critical edition of the Florence and Milan manuscripts.

7 The last preceding problem which is shared by the Florence and the Vatican manuscripts deals with the roof of a square building; here, the Florence version has deleted an outer framework for the problem, "Uno cictadino vole fare over a facto fare uno palagio come tu vedi qui da parte designato". After the problem, it inserts a kind of note to the preceding calculation, explaining how to find the root of 101 (which is told to equal $10+\frac{1}{2\cdot 10}$); the reason for the note is probably that $\sqrt{569}$ has just been found in the same way (it is true that the rule has already been enunciated in general terms and illustrated with examples on fols. 39^v–40^r, where it is_told to give the true root or the closest approximation).

In the Vatican version, the note is not present, nor would it be appropriate: here, $\sqrt{569}$ is found by the erroneous second-order approximation $23+\frac{20}{23}-\frac{400}{529}$, obviously derived from the observation that $(23+\frac{20}{23})^2 = 569+\frac{400}{529}$. (In this ms, the first-order approximation is explained on fols. 32^v–33^v, without the erroneous statement that this may be the true root).

After the algebra section, the Vatican manuscript goes on exactly as the Florence version (apart from differences in the wording), first with an invocation of God and next with a preamble to the ensuing treatment of alligation.

sequence might be interpolations; we shall return with further arguments that the algebra must in any case be earlier than Paolo Gherardi's algebra from 1328, so far considered the earliest vernacular algebra. For the moment I shall just observe that a number of characteristic phrases are shared by the algebra section of the Vatican manuscript and its other sections, which suggest common authorship or, at the very least, much closer affinity between the two components than between any of them and any other work I have looked at from the period. The same conclusion is suggested by the shared use of the *compagnia* as an abstract representation for proportional partitioning.[8] The Vatican version might still be believed to be a later revision of a work whose original is copied in the Florence and Milan manuscripts (whether due to Jacopo or some otter hand); close comparison of the Vatican and Florence manuscripts show, however, that the latter instead contains a revised version of an original that was close to the Vatican version, and which which is also likely to have contained the algebra and the final problem collection.[9] All in all, there are no serious reasons to doubt that the algebra section belongs to the original work, nor

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[8] The model is used to determine the shares of an inheritance (Florence ms., Fol. 28[r]–28[v], ed. [Simi 1995: 19], Vatican ms. Fol. 23[v]–24[r]); in the algebra section (Vatican ms., Fol. 37[r], [3] below); in several alligation problems in the Vatican ms.; and in a problem where the participants in a *compagnia* do not enter at the same time, and where a different company is imagined where investments are multiplied by duration (Vatican ms, Fol. 50[v]–51[r], below, [A1]). The latter type is not uncommon, but it constitutes only a modest generalization of the basic structure of the company; an *abbaco* treatise from Lucca (to whose closeness to the "Vatican Jacopo" we shall return) also uses it in an alligation problem and treats of an analogous inheritance problem under the general heading of the company [ed. Arrighi 1973: 97, 136]; but apart from this work, I have noticed the genuine use of the company as a functionally abstract representation in no place outside the Jacopo manuscripts.

[9] I intend to publish this evidence elsewhere, since the argument presupposes much textual analysis. It turns out, among other things, that the Florence text contains a number of inconsistencies which can only be explained as traces of an inconsistent revision. The Vatican text as a whole, moreover, is very consistent both in linguistic, discursive and pedagogical style and in mathematical approach. If it had merged an original shorter treatise with secondary accretions, the whole text would have had to go through a process of secondary harmonization – but this would certainly not have left the original part unchanged but made it diverge just as much as the Florence text from the common origin. Finally, the final problem collection turns out not to contain a single overlap with the preceding text but cross-referenced variations and supplements. This is certainly not what we find in those *abbaco* texts which are known to be composites.

to reject its date.

It should be noticed that being “the earliest European vernacular algebra so far known” does not entail being really first – not only for logical reasons but because of actual though indirect evidence. A strange change of terminology in Leonardo Fibonacci’s *Liber abaci* suggest that another vernacular treatise may have existed before 1228. Fibonacci’s section on *algebra et almuchabala* [ed. Boncompagni 1857a: 406–459] makes use of the standard *res-census*-terminology introduced by Gherardo da Cremona as translations of Arabic *šayʾ* (“thing”) and *māl* (“possession”). Between pp. 427 and 446, however, the unknown is designated *avere* in 13 problems, that is, by an Italian vernacular translation of *māl*. In the first three cases (pp. 427, 430, 433) this quantity is identified with the *res*, which allows Leonardo to speak of its square as *māl*; at a pinch, this might have been a reason to choose an unconventional and self-invented translation of an Arabic *māl*. The explanation is not too likely, since Leonardo could just as well have spoken of the unknown as *quidam numerus* – that is what he does in the preceding problem on p. 427; have referred to the square on the unknown *māl* as *census census*, as on p. 439; or have relabelled an original *census* as *res*, as done on p. 422 (*pone pro ipso censu rem*); but it remains at least a faint possibility that he tried out still another strategem to circumvent the same problem. In the ten instances between p. 442 and p. 446, however, the *avere* is immediately identified with a *census* – there *is* no problem to circumvent. In view of this it seems extremely unlikely that Leonardo should have introduced the vernacular term (a thing he does nowhere else) *unless it was already in use*. Somebody must have spoken or written about second-degree algebra in *volgare* at least before Leonardo prepared the second edition of his treatise in 1228 and “added certain things that are needed and cut out others that are superfluous” (p. 1; trans. JH).[10]

A possible candidate exists for this precursor: in the fourteenth and sixteenth centuries, respectively, Rafaele Canacci and Francesco Ghaligai claim that Guglielmo de Lunis made an Italian translation of al-Khwārizmī.[11] Since Guglielmo may have been connected to Frederick II’s court

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[10] I used Boncompagni’s edition, which is based on a single manuscript (Florence, Conv. sopp. C. 1. 2616). As is obvious from the preceding description, however, the structure of the problems in which the term *avere* turns up shows that it cannot be a simple translation of an original *census* due to a copyist.

[11] See [Libri 1838: II, 45]; cf. further discussion in [Karpinski 1910: 210*f*] and [Kaunzner 1985: 6, 11*f*].

in Naples, a translation made before 1228 could well have been exploited by Leonardo, who prepared his second version for the same courtly circle.

However, since all fourteenth-century and later *abbaco* algebras use the *censo-cosa* terminology, they are obviously not in debt to this early vernacular treatise; apart from its possible (but except for the terminology wholly unsubstantiated) influence on Leonardo, it seems to represent a dead end.

Edition, translation and mathematical commentary

In the following text edition, abbreviations are dissolved,[12] word separation (uneven in the ms[13]) and capitalization is normalized, interpunctuation, diacritics and apostrophees are added in agreement with the standard established by Gino Arrighi; similarly, a distinction between *u* and *v* has been introduced.[14] To the extent it seemed reasonable, the interpunctuation follows what is suggested by the interpunctuation and the capitalizations of the manuscript. Omitted words and passages are inserted in pointed brackets, as ⟨...⟩, superfluous words and passages appear as {...}, and editorial commentaries as [...].

The English translation attempts to keep very close to the text, and to render always in the same way the same phrase or term when used in the same function, even in cases where this implies some awkwardness. The purpose is in part to reflect the medium which Jacopo had at his disposal – a vernacular which was not yet fashioned as an adequate tool for a technical discourse; in part it is to render that imprecision of the conceptual structure which was a consequence of the character of the language (but which probably had other roots as well). However, the principle is not followed to the pedantic extremes which I have felt it necessary to accept

[12] It is noteworthy that the central words *censo*, *cosa*, *cubo*, *radice*, *più* and *meno* are never abbreviated. Metrological and monetary units are mostly abbreviated, but written in full sufficiently often to allow indubitable expansion.

[13] In cases where standard Italian has two words but the manuscript begins the second with a double consonant, thus indicating pronunciation as one word, I have used an apostrophee instead of full separation – thus “sì’tti” instead of “sì tti”.

[14] In contrast, I do not distinguish *i* from *j*. Both occur in the manuscript, but only as mere graphic variants with no functional implications; similar variants exist for both *s*, *u* and *r*.

when rendering Babylonian mathematical texts; loose as it is, the conceptual structure of Jacopo’s mathematics is after all not very different from ours, even though it is still *in statu nascendi*.

Censo, *cubo* and *censo de censi* are left untranslated in order to emphasize their role as technical terms within a particular algebraic representation that should not automatically be confounded with our unlimited sequence of powers. *Cosa*, whose everyday connotations could not be ignored by Jacopo and his readers, is translated accordingly as “thing”.

In the mathematical commentary, t stands for *thing* and C for *censo*, K for *cubo* and CC for *censo de censi* (that is, $C = t^2$, $K = t^3$, $CC = t^4$). a, b, d, etc. stand for unknown numbers or quantities, p for an unknown ratio. Greek letters stand for given numbers. “Positing” is indicated “:=”.

[1] ***(Fol 36^v)*** Quando le cose sonno eguali al numero, si vole partire el numero nelle cose, et quello che ne vene si è numero. Et cotanto vale la cosa.

[1] ***(Fol 36^v)*** When the things are equal to the number, one shall divide the number by the things, and what results from it is a number. And as much is the thing.

Mathematical commentary: $\alpha t = \beta \;\Rightarrow\; t = \frac{\beta}{\alpha}$.

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[2] Pongoti assempro ala dicta ragione. Et vo’ dire così: Fammi de 10 doy parti, che partita la magiore nella minore ne vengha 100. Fa così: poni che la magiore parte fosse una cosa. Adunqua la minore serà lo rimanente infino in 10 che serà 10 meno una cosa. Et così abiamo facto de dece doy parti, che la magiore sia una cosa, et la minore sia 10 meno una cosa. Ora si vole partire la magiore nella minore, cioè una cosa in 10 meno una cosa; che ne dè venire 100. Et però dè multiplicare 100 via 10 meno una cosa; fa 1000 meno 100 cose, che s’aoguaglino a una cosa. Ora ristora ciascheuna parte, cioè de giungere 100 cose che sonno meno a ciascheuna parte. Arai che 101 cosa sonno iguali a 1000 numeri. Et però se vole partire li numeri nelle cose, cioè 1000 numeri in 101 cosa, che ne vene 9 et $^{92}/_{101}$ [*sic*], et cotanto vale la cosa. Et noi porremo che la magiore parte fusse una cosa, dunqua vale, et dirremo che la magiore parte de 10 sia 9 et $^{91}/_{101}$. Et la seconda serà el resto infino in 10, che serà $^{10}/_{101}$. Et abiamo che la magior parte de 10 serà 9 e $^{91}/_{101}$, et la minore $^{10}/_{101}$. Ora parti 9 et $^{91}/_{101}$ in $^{10}/_{101}$, che ne vene appunto 100; et sta bene. Et così se fa le simili ragioni.

[2] I propose to you an example of the said computation. And I will say thus: Make two parts of 10 for me, so that when the larger is divided by the lesser, 100 results from it. Do thus: Posit that the larger part was a thing. Then the lesser will be the remainder until 10, which will be 10 less a thing. And thus we have made two parts of ten, of which the larger be a thing, and the lesser be 10 less a thing. Now the larger shall be divided by the lesser, that is a thing by 10 less a thing; from which should result 100. And therefore one should multiply 100 times 10 less a thing; it makes 1000 less 100 things, which equal one thing. Now restore each part, that is, to join 100 things which are less to each part. You will get that 101 thing are equal 1000 in numbers. And therefore one shall divide the numbers by the things, that is 1000 in numbers by 101 thing, from which results 9 and $^{92}/_{101}$ [*sic*], and as much is the thing. And we posited that the larger part was a thing, which is then what it will be, and we shall say that the larger part of 10 is 9 and $^{91}/_{101}$. And the second will be the rest until 10, which will be $^{10}/_{101}$. And we have that the larger part of 10 will be 9 and $^{91}/_{101}$, and the lesser $^{10}/_{101}$. Now divide 9 and $^{91}/_{101}$ by $^{10}/_{101}$, from which results precisely 100. And it goes well. And thus one makes the similar computations.

Mathematical commentary: The problem can be expressed

$$10 = a+b \ , \ \ a/b = 100 \ .$$

Positing $a := t$ we have $b = 10–t$, and thus

$$t/(10–t) = 100 \ , \ \ t = 1000–100t \ , \ \ 101t = 1000 \ , \ \ t = 1000/101 = 9^{91}/_{101} \ .$$

The solution is thus

$$a = 9^{91}/_{101} \ , \ \ b = 10–9^{91}/_{101} = {}^{10}/_{101} \ .$$

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[3] Anco' ti voglio porre uno altro assempro, et vo' dire chosì. E sonno tre conpagni che ànno guadagnato 30 libre. El primo conpagnio misse 10 libre. El secondo misse 20 libre. El terzo misse tanto che de questo guadagnio gle tocchò 15 libre. Vo' sapere quanto misse el terzo conpagnio, et quanto toccha per uno de guadagnio de quelli altri doy conpagni. Fa così: se noi vogliamo sapere quanto misse el terzo conpagnio, poni che el terzo mettesse una chosa. Appresso se vole raccoglere quello che mise el primo et el secondo, cioè libre 10 et libre 20, che sonno 30. Et arai che sonno tre conpagni, che el primo mette in conpagnia 10 libre; el secondo mette 20 libre; el terzo mette una chosa. Sì che el corpo dela conpagnia è 30 libre et una cosa. Et ànno guadagnato 30 libre. ***(Fol 37r)*** Ora se noi vogliamo sapere quanto toccha al terzo conpagnio de questo guadagnio, che abbiamo posto che mettesse una cosa, sì'tti convene multiplicare una chosa via quello che egli ànno guadagniato, et partire in tucto el corpo dela conpagnia. Et però abbiamo a multiplicare 30 via una cosa. Fa 30 cose, le quale te convene partire nel corpo dela conpagnia, cioè per 30 et una cosa, et quello che ne vene cotanto toccha al terzo conpagnio. Et questo non ci fa bisogno partire perché noi sappiamo che glie ne toccha 15 libre. Et però multiplicha 15 via 30 et una cosa. Fanno 450 et 15 cose. Dunqua 450 numeri et 15 cose s'aoguagliano a 30 cose. Ristora ciascheuna parte, cioè de cavare de ciascheuna parte 15 cose. Et arai che 15 cose se aoguagliano a 450 numeri. Et però devi partire li numeri nelle cose, cioè 450 in 15, che ne vene 30. Et cotanto vale la chosa. Et noi ponemo che el terzo conpagnio mettesse una cosa, sì che vene ad avere messo 30 libre. El secondo 20 libre. El primo 10 libre. Et se volesse sapere quanto ne toccha al primo et al secondo, si cava di 30 libre 15 che'nne toccha al terzo. Restano 15 libre. Et dirrai che sonno 2 conpagni che ànno guadagnato 15 libre. Et el primo misse 10 libre. Et el secondo misse 20 libre. Quanto ne toccha per uno? Fa così et di', 20 libre et 10 libre sonno 30 libre, et questo è el corpo dela conpagnia. Ora multiplicha per lu primo, che mise 10 libre, 10 via 15 che ànno guadagniato; fanno 150. Parti in 30, che ne vene 5 libre. Et cotanto ne toccha al primo. Et poi per lo secondo multiplicha 20 via 15, che fa 300 libre. Parti in 30 che ne vene 10 libre, et cotanto toccha al secondo conpagnio. Et è facta, et sta bene. Et così se fanno le simili ragioni.

[3] Again, I shall propose to you another example, and I shall say thus. There are three companions, who have gained 30 *libre*. The first companion put in 10 *libre*. The second put in 20 *libre*. The third put in so much that 15 *libre* of this gain went to him. I want to know how much the third companion put in, and how much of the gain goes to each of the other two companions. Do thus: if we want to know how much the third put in, posit that the third put in a thing. Next one shall collect that which the first and the second put in, that is 10 *libre* and 20 *libre*, which are 30 *libre*. And you will get that there are three companions, of whom the first put in the company 10 *libre*; the second put in 20 *libre*; the third put in a thing. So that the principal of the company is 30 *libre* and a thing. And they have gained 30 *libre*. ***(Fol 37^r)*** Now if we want to know how much of this gain goes to the third companion, when we have posited that he put in a thing, then you ought to multiply a thing times that which they have gained, and divide by the total principal of the company. And therefore we have to multiply 30 times a thing. It makes 30 things, which you ought to divide by the principal of the company, that is by 30 and a thing, and what results from it, as much goes to the third companion. And this we do not need to divide, because we know that 15 *libre* of it goes to him. And therefore multiply 15 times 30 and a thing. It makes 450 and 15 things. Then 450 in numbers and 15 things equal 30 things. Restore each part, that is to remove from each part 15 things.[15] And

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[15] This use of "restoration" (which recurs in [9], [13] and [14] and is hence no simple slip of the pen) is a noteworthy deviation from al-Khwārizmīan norms, according to which removal of a positive quantity is termed "opposition", and "restoration" is reserved for addition to both sides of the equation, as in Jacopo's [2]. Cf. note 47.

you will get that 15 things equal 450 in number. And therefore you should divide the numbers by the things, that is 450 by 15, from which results 30. And as much is the thing. And we posited that the third companion put in a thing, so that he turns out to have put in 30 *libre*. The second 20 *libre*. The first 10 *libre*. And if one wants to know how much of it goes to the first and to the second, from 30 *libre* one removes 15 of them which go to the third. 15 *libre* remain. And you will say that there are 2 companions who have gained 15 *libre*. And the first put in 10 *libre*. And the second put in 20 *libre*. How much of it goes to each one? Do thus, and say, 20 *libre* and 10 *libre* are 30 *libre*, and this is the principal of the company. Now multiply for the first, who put in 10 *libre*, 10 times 15 which they have gained; it makes 150. Divide by 30, from which results 5 *libre*. And as much goes to the first. And then for the second, multiply 20 times 15, which makes 300 *libre*. Divide by 30, from which results 10 *libre*, and as much goes to the second companion. And it is done, and it goes well. And thus the similar computations are made.

Mathematical commentary: The investments of the three companions are a, b and c, and the rate of profit is p; the unit is the *libra*. Then

$$pa+pb+pc = 30\ ,\quad a = 10\ ,\quad b = 20\ ,\quad pc = 15\ .$$

Positing $c := t$ we thus have

$$p\cdot(a+b+t) = 30 \quad\text{or}\quad p\cdot(30+t) = 30\ .$$

By the rule of three, the third companion gains

$$pc = t\cdot\frac{30}{30+t}\ ,$$

and since this gain is 15,

$$15\cdot(30+t) = 30t \quad\text{or}\quad 450+15t = 30t \quad\text{or}\quad 15t = 450\ .$$

Hence $t = c = 30$.

Then the part of the gain which falls to the first two companions is 30–15 = 15, while their investment is 10+20 = 30; this is divided proportionally by the rule of three,

$$pa = 15\cdot\frac{10}{30} = 5\ ,\quad pb = 15\cdot\frac{20}{30} = 10\ .$$

It is noteworthy that the latter computation is performed within the framework of a fictitious company. In the present case this fictitious company is still part of the "real" company which the problem treats of; however, in problem [A1] of the appendix (p. 38) we shall encounter a wholly fictitious company; it is also used in a heritage problem on fol. 24^{v} and in an alligation problem on fol 48^{r-v}. The role of the "company" as a *functionally abstract representation of the mathematical structure of proportional division* is therefore subject to no doubt.

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[4] Quando li censi sonno oguali al numero, si vole partire el numero per li censi. Et la radice de quello che ne vene vale la cosa.

[4] When the *censi* are equal to the number, one shall divide the number by the *censi*. And the root of that which results from it is the thing.

Mathematical commentary: $\alpha C = \beta \;\Rightarrow\; t = \sqrt{\frac{\beta}{\alpha}}$.

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[5] ***(Fol 37^v)*** Assemplo ala dicta regola. Et vo' dire chosì: Trovame doi numeri che siano in propositione sì come è 2 de 3; et multiplicato ciascheuno per se medesimo, et tracta l'una multiplicatione dell'altra, remangha 20. Vo' sapere qual' numeri sonno questi. Fo così, et poni che l'uno numero fosse 2 chose et l'altro fosse 3 cose. Et bene sonno in propositione sì come sonno 2 et 3. Appresso si vole multiplicare li numeri, ciascheuno per se medesemo. Et cavare l'una multiplicatione dell'altra. Et deve remanere 20. Et però multiplicha ciascheuno per se, et di': duo cose via 2 cose fanno 4 censi. Et tre cose via 3 cose fanno 9 censi. Ora cava l'una multipricatione dell'altra, cioè 4 de 9. Resta 5 censi, i quali s'aoguagliano a 20 numeri. Et noi diciamo che se voli partire[16] li numeri nelli censi, sì che se vole partire 20 numeri in 5 censi; che ne vene 4 numeri et cotanto vale la cosa, cioè la sua radice che è 2.[17] Dicemo che fosse el primo numero 2 cose et el secondo 3 cose. Però vedi chiaro che 2 cose vagliono 4 numeri. Et 3 cose 6 numeri. Et così te dicho che questi numeri sonno l'uno 4 et l'altro 6. Et tal parte è 4 de 6 qual 2 de 3. Ora se la voi provare, multipricha 6 via 6, fa 36. Et multipricha 4 via 4, fa 16. Tray 16 de 36. Resta 20, et sta bene. Et chosì se fano tucte le simiglianti ragioni, cioè secondo questa regola.

[5] ***(Fol 37^v)*** Example of the said rule. And I shall say thus: find me two numbers that are in the same proportion as is 2 of 3: and when each of them is multiplied by itself, and one multiplication is detracted from the other, 20 remains. I want to know which are these numbers. Do thus, and posit that one number was 2 things and the other was 3 things. And they are well in the same proportion as are 2 and 3. Next one shall multiply the numbers, each by itself. And remove one multiplication from the other. And 20 shall remain. And therefore multiply each by itself, and say: two things times 2 things make 4 *censi*. And three things times 3 things make 9 *censi.* Now remove one multiplication

[16] This word was forgotten during copying and inserted between the lines.

[17] First written «20», the «0» then crossed over.

from the other, that is, 4 from 9. 5 *censi* remains, which equal 20 in numbers. And we say that one shall divide the numbers by the *censi*, so that one shall divide 20 in numbers by 5 *censi*; from which results 4 in numbers, and as much is the thing, that is its root, which is 2. We said that the first number was 2 things and the second 3 things. Therefore you see clearly that 2 things are 4 in numbers. And three things 6 in numbers. And thus I say to you that these are 4, one, and 6, the other. And such part is 4 of 6 as 2 of 3. Now if you want to verify it, multiply 6 times 6, it makes 36. And multiply 4 times 4, it makes 16. Detract 16 from 36. 20 remains, and it goes well. And thus all the similar computations are made, that is, according to this rule.

Mathematical commentary: The problem can be expressed

$$a:b = 2:3\ ,\ \ b^2–a^2 = 20\ .$$

Positing $a:= 2t$, $b:= 3t$ we get

$$9C–4C = 5C = 20 \ \text{ or } \ C = 4 \ \text{ or } \ t = \sqrt{4} = 2\ ,$$

and thus

$$a = 2t = 4\ ,\ \ b = 3t = 6\ .$$

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[6] Quando li censi sonno oguali ale chose, se vole partire le cose per li censi, et quello che ne vene si è numero. Et cotanto vale la cosa.

[6] When the *censi* are equal to the things, one shall divide the things by the *censi*, and that which results from it is a number. And as much is the thing.

Mathematical commentary: $\alpha C = \beta t \ \Rightarrow\ t = \frac{\beta}{\alpha}\ .$

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[7] Assemplo ala dicta regola: Trovami 2 numeri che siano in propositione sì como è 4 de 9. Et multiprichato l'uno contra l'altro faccia quanto ragionti inseme. ***(Fol 38^r)*** Vo' sapere qual' numeri sonno questi. Fa così: poni che l'uno numero sia 4 cose. Et l'altro numero sia 9 chose. Et bene è in propositione come è 4 a 9. Adunque l'uno numero è 4 chose. Et l'altro è 9 chose. Et noi diciamo che vogliamo fare tanto multiprichati l'uno contra a l'altro quanto raggionti inseme. Et però multipricha 4 cose via 9 cose, fanno 36 censi. Et aggiongi inseme 4 e 9 cose, fanno 13 cose, et ài che 36 censi sonno oguali a 13 cose. Et però parti 13 cose in 36 numeri; che ne vene $^{13}/_{36}$ de numero, et cotanto vale la cosa. Ora noi ponemo che l'uno numero fusse 4 cose. Però multipricha 4 via {erasure} $^{13}/_{36}$, che fa $^{52}/_{36}$, che sonno I e $^{4}/_{9}$. Et cotanto è l'uno numero. Et ponemo che l'altro numero fusse 9; però multipricha 9 via $^{13}/_{36}$, che fa $^{117}/_{36}$, che sonno 3 et $^{1}/_{4}$. Et cotanto è'll'altro numero. Ora se la voli provare, si multipricha I et $^4/_9$ via 3 et $^1/_4$, che fanno 4 et $^{25}/_{36}$. Ora agiongi inseme li dicti numeriche, che fanno quello medesimo. Et sta bene. Et così se fanno le simili ragioni.

[7] Example of the said rule: Find me 2 numbers that are in the same proportion as is 4 of 9. And when one is multiplied against the other, it makes as much as when they are joined together. ***(Fol 38ʳ)*** I want to know which are these numbers. Do thus: posit that one number be 4 things. And the other number be 9 things. And they are well in the same proportion as is 4 to 9. Then one number is 4 things. And the other is 9 things. And we say that we want them to make as much when they are multiplied one against the other as when they are joined together. And therefore multiply 4 things times 9 things, it makes 36 *censi*. And join together 4 and 9 things, they make 13 things, and you have that 36 *censi* are equal to 13 things. And therefore divide 13 things by 36 in numbers; from which results $^{13}/_{36}$ in number, and as much is the thing. Now we posited that one number was 4 things. Therefore multiply 4 times $^{13}/_{36}$, which makes $^{52}/_{36}$, which are I and $^4/_9$. And as much is one of the numbers. And we posited that the other number was 9: therefore multiply 9 times $^{13}/_{36}$, which makes $^{117}/_{36}$, which are 3 and $^1/_4$. And as much is the other number. Now if you want to verify it, one multiplies I and $^4/_9$ times 3 and $^1/_4$, which make 4 and $^{35}/_{36}$. Now adjoin together the said numbers, which make the very same. And it goes well. And thus the similar computations are made.

Mathematical commentary: The problem can be expressed

$$a:b = 4:9\ ,\quad a\cdot b = a+b\ .$$

Positing $a := 4t$, $b := 9t$ we have

$$4t\cdot 9t = 4t+9t \quad\text{or}\quad 36C = 13t\ ,$$

and hence $t = {}^{13}/_{36}$.

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[8] Quando li censi et le cose sonno oguali al numero se vole partire neli censi, et poi demezzare le cose et multiprichare per se medesimo et giungere sopra al numero. Et la radice dela somma meno el dimezzamento dele cose vale la cosa.

[8] When the *censi* and the things are equal to the number, one shall divide by the *censi*, and then halve the things and multiply by itself and join above the number. And the root of the sum less the halving of the things is the thing.

Mathematical commentary: $\alpha C+\beta t = \gamma \;\Rightarrow\; t = \sqrt{\frac{\gamma}{\alpha} + (\frac{\beta}{\alpha}:2)^2} - (\frac{\beta}{\alpha}:2)$.

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[9] Assemplo[18] ala dicta Regola. Et vo' dire chosì: uno presto a un'altro 100 libre al termine de 2 anni a fare capo d'anno. Et quando vene ala fine de 2 anni et quegli glie rendi libre 150. Vo' sapere ad ***(Fol 38^v^)*** que ragione fo pres⟨ta⟩ta la libra el mese. Fa così: pone che fusse prestata a una cosa el mese de denaro, sì che vene a valere l'anno la libra 12 cose de denaro, che 12 cose de denaro sonno el vigensimo de una libra, sì che la libra vale l'anno $^1/_{20}$ ⟨de cosa⟩ de una libra. Et però di' così: se la libra vale {deleted word} l'anno $^1/_{20}$ de una libra, que varranno 100 libre? Multipricha 100 via $^1/_{20}$. Fa $^{100}/_{20}$, che sonno 5 cose. Agiongi sopra a 100 libre. Fanno 100 libre e 5 cose per uno anno. Ora se voli sapere per lo secondo anno, multipricha 100 libre et 5 cose via $^1/_{20}$ de cosa. Fanno 5 cosa et $^1/_4$ censo, le quali se vogliono agiongere a 100 libre et 5 cose, che fanno 100 libre e 10 cose et $^1/_4$ censo. Et cotanto sonno le 100 libre in 2 anni, tra merito et capitale. Et essendo prestata la libra el mese a una cosa. Et noi sappiamo de certo che le 100 libre ànno guadagniato in 2 anni 50 libre. Sì che le 150 libre vagliono le 100 libre e 10 cose et $^1/_4$ censo. Sì che le 100 libre, 10 cose et $^1/_4$ censo sonno oguali a 150 libre. Ristora ciascheuna parte, cioè cavare 100 libre de ogni parte, et arai che 10 cose et $^1/_4$ censo sonno oguali a 50. Ora fa sì como dice la nostra regola, cioè de arrechare a uno censo, cioè de partire in $^1/_4$ censo, et arai che I censo et 40 cose sonno oguali a 200 numeri. Ora demezza le cose. Sonno 20. Multipricha per se medesemo, fa 400; aggiongi sopra li numeri, fanno 600. Trova la sua radice, la quale è sorda, cioè, che è manifisto, de non avere radice appunto, et cotanto dirremo che vaglia la cosa, cioè la radice di 600 meno 20, cioè el dimezzamento dele cose. Et noi ponemo che fusse prestata la libra el mese a una cosa de denaro, dunque dirremo che fusse prestata la libra el mese a denari, la radice di 600 meno 20 denari. Et sta bene. Et così se fanno le simiglianti ragioni.

[9] Example of the said rule. And I shall say thus: one lends to another 100 *libre* at the term of 2 years, to do at the end of year.[19] And when the two years came to an end he gave back to him 150 *libre*. I want to know at ***(Fol 38^v^)*** which rate the *libra* was lent a month. Do thus: posit that it was lent at one thing in *denaro* a month, so that the *libra* turns out to be worth 12 things in *denaro* a year, which 12 things in *denaro* are the twentieth of a *libra*, so that the *libra* is worth $^1/_{20}$ ⟨thing⟩ of a *libra* a year. And therefore say thus: if the *libra* is worth $^1/_{20}$ of a *libra* a year, what will 100 *libre be* worth? Multiply 100 times $^1/_{20}$. It makes $^{100}/_{20}$, which are 5 things. Adjoin above 100 *libre*. They make 100

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[18] Corrected from «assempro» – unless the correction went the other way, the ink is the same. In any case the copyist is seen to be conscious of orthography, and probably to follow his original.

[19] That is, at composite interest, computed yearly.

libre and 5 things for one year. Now if you want to know for the second year, multiply 100 *libre* and 5 things times $\frac{1}{20}$ of thing. They make 5 thing and $\frac{1}{4}$ *censo*, which are to be adjoined to 100 *libre* and 5 things, which make 100 *libre* and 10 things and $\frac{1}{4}$ *censo*. And as much are the 100 *libre* in 2 years, interest and capital together. And being lent the *libra* at one thing a month. And we know for sure that the 100 *libre* have gained 50 *libre* in 2 years. So that the 150 *libre* are the 100 *libre* and 10 things and $\frac{1}{4}$ *censo*. So that the 100 *libre*, 10 things and $\frac{1}{4}$ *censo* are equal to 150 *libre*. Restore each part, that is, to remove 100 *libre* from each part, and you will get that 10 things and $\frac{1}{4}$ *censo* are equal to 50. Now do so as our rule says, that is, to bring to one *censo*, that is, to divide by $\frac{1}{4}$ *censo*, and you will get that I *censo* and 40 things are equal to 200 in numbers. Now halve the things. They are 20. Multiply by itself, it makes 400; adjoin above the numbers, they make 600. Find its root, which is surd, that is, as it is manifest, to have no precise root, and as much will we say that the thing is, that is the root of 600 less 20, that is the halving of the things. And we posited that the *libra* was lent at one thing of *denaro* a month, then we will say that the libra was lent at the root of 600 less 20 *denari* a month. And it goes well. And thus the similar computations are made.

Mathematical commentary: The loan is 100 £, 1 £ = 20s, 1s = 12d. The monthly interest is posited to be td, for which reason the yearly interest on 1 £ is 12 d = $^1/_{20}t$ £. After 1 year, the 100 £ are therefore worth $(100+{}^{100}/_{20}t)$ £ = $(100+5t)$ £, and the interest of the second year will be $(100+5t)\cdot{}^1/_{20}t$ £ = $(5t+{}^1/_4C)$ £. Therefore 150 £ = 100 £ + $10t+{}^1/_4C$. “Restoring”, that is, subtracting 100 £, we get

$$10t+{}^1/_4C = 50 \quad\text{or}\quad C+40t = 200\ .$$

According to the rule, $t = \sqrt{20\cdot 20+200} - 20 = \sqrt{600} - 20$, which will be the monthly interest on 1 £.

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[10] ***(Fol 39^r^)*** E sonno due homini che ànno denari. Dice el primo al secondo: Se tu me dessi 14 de toi denari, che io li racchozzasse co’ mey, io arei 4 cotanti de te. Dice el secondo al primo: se tu me desse la radice de toy denari, io arei denari 30. Vo’ sapere quanto aveva ciascheuno homo. Fa chosì: poniamo che’l primo homo avesse I censo. Et egli adimanda 14 al secondo, sì che verrà ad avere I censo e 14. Et dice de avere 4 cotanti de lui. Dunque convene che rimangha al secondo el $^1/_4$ ⟨censo⟩ e $3^1/_2$. Dunque nanzi che ’l secondo desse nulla al primo sì n’aveva egli $17^1/_2$ et $^1/_4$ censo. Et così abiamo che ’l primo vene ad avere uno censo. Et el secondo $17^1/_2$ et $^1/_4$ censo. Et poi domanda el secondo al primo la radice de soi denari, cioè de I censo, che è una chosa, la quale se vole agiongere a $^1/_4$ censo e $17^1/_2$. Et in verità fa $^1/_4$ censo et una chosa et $17^1/_2$, et con questo dice che de avere 30. Adunque abiamo che $^1/_4$ censo et una chosa et $17^1/_2$ numeri sonno oguali a 30. Ristora ciascheuna parte, cioè tray $17^1/_2$ de ogni una parte. Et arai che $^1/_4$ censo et una chosa sonno oguali a $12^1/_2$ numero. Dei partire per ɪ [*sic*] censo et arai che uno censo e 4 chose sonno oguali a 50. Ora demezza le cose, sonno 2. Multipricha per se medesimo, fa 4. Aggiongi sopra ai numeri a 54, et de questo trova la sua radice, et cotanto vale la cosa meno el dimezzamento dele cose, cioè 2. Et noi ponemo el primo avesse uno censo. Et però ti convene sapere que vale el censo. Et però di': multiprichare radice de 54 meno 2 via radice de 54 meno 2. Et cotanto varrà el censo. Che in verità, radice de 54 meno 2 via radice de 54 meno 2, fa 58 meno radice de [ ][20] et abbiamo che vale el censo 58 meno radice [ ]. Et noi ponemo avesse el primo uno censo. Dunque vene ad avere 58 meno radice de [ ]. ⟨Ora sappi el secondo, che ponesti ch'avesse $^1/_4$ censo e $17^1/_2$ numeri. Adunque piglia el $^1/_4$ de 58 meno radice de 864⟩[21] ch'è $14^1/_2$ meno radice de 54, sopra el quale vi giongi $17^1/_2$; fanno 32 mino la radice de 54. Et così abiamo che el primo à 58 meno la radice de [ ]. Et el secondo homo à 32 meno radice de 54. Et è facta. Et così se fanno le simiglanti ragioni.

[10] ***(Fol 39^r)*** And there are two men that have *denari*. The first says to the second: If you gave me 14 of your *denari*, and I threw them together with mine, I would have 4 times as much as you. The second says to the first: if you gave me the root of your *denari*, I would have 30 *denari*. I want to know how much each man had. Do thus: we posit that the first man had ɪ *censo*. And he asks for 14 from the second, so that he will come to have ɪ *censo* and 14. And he says to have 4 times as much as him. Then to the second the $\frac{1}{4}$ ⟨*censo*⟩ and $3\frac{1}{2}$ ought to remain. Then, before the second gave anything to the first, he had $17\frac{1}{2}$ and $\frac{1}{4}$ *censo*. And thus we have that the first turns out to have one *censo*. And the second $17\frac{1}{2}$ and $\frac{1}{4}$ *censo*. And then the second asks the first for the root of his *denari*, that is of ɪ *censo*, which is a thing, which one shall adjoin to $\frac{1}{4}$ *censo* and $17\frac{1}{2}$. And truly it makes $\frac{1}{4}$ *censo* and one thing and $17\frac{1}{2}$, and with this he says to have 30. Then we have that $\frac{1}{4}$ *censo* and one thing and $17\frac{1}{2}$ in numbers are equal to 30. Restore each part, that is, detract $17\frac{1}{2}$ from each part. And you will get that $\frac{1}{4}$ *censo* and one thing are equal to $12\frac{1}{2}$ in number. You shall divide by 1 [*sic*] *censo* and you will get that one *censo* and 4 things are equal to 50. Now halve the things, they are 2. Multiply by itself, it makes 4. Adjoin above the numbers, to 54, and of this find its root, and as much is the thing less the halving of the things, that is, 2. And we posited that the first had a *censo*. And therefore you ought to know what the *censo* is. And therefore say: to multiply root of 54 less 2 times root of 54 less 2. And as much will the *censo* be. And truly, root of 54 less 2 times root of 54 less 2, makes

[20] Instead of «864», the ms leaves open c. 2 cm. In the margin the copyist writes the commentary «così stava nel'originale spatii».

[21] Lacuna completed according to *Trattato dell'alcibra amuchabile*, ed. [Simi 1994: 25], but in agreement with Jacopo's normal orthography (cf. below, p. 50).

58 less root of [   ], and we have that the *censo* is 58 less root [   ]. And we posited that the first had a *censo*. Then he turns out to have 58 less root of [   ]. ⟨Now know the second, of whom you posited that he had $\frac{1}{4}$ *censo* e $17\frac{1}{2}$ in numbers. Then take the $\frac{1}{4}$ of 58 less root of 864⟩, which is $14\frac{1}{2}$ less root of 54, above which join $17\frac{1}{2}$; they make 32 less the root of 54. And so we have that the first has 58 less the root of [   ]. And the second man has 32 less root of 54. And it is done. And thus the similar computations are made.

Mathematical commentary: If the two possessions are a and b, the problem is

$$a+14 = 4\cdot(b-14)\ ,\quad b+\sqrt{a} = 30\ .$$

Positing $a := C$, this gives

$$b-14 = {}^1\!/_4 C+3\,{}^1\!/_2 \quad\text{or}\quad b = 17\,{}^1\!/_2+{}^1\!/_4 C\ .$$

Further, since $\sqrt{a} = \sqrt{C} = t$,

$$b+\sqrt{a} = 17\,{}^1\!/_2+{}^1\!/_4 C+t = 30 \quad\text{or}\quad {}^1\!/_4 C+t+17\,{}^1\!/_2 = 30\ .$$

“Restoring” we get

$${}^1\!/_4 C+t = 12\,{}^1\!/_2 \quad\text{or}\quad C+4t = 50\ ,$$

which according to the rule gives

$$t = \sqrt{2\cdot 2+50}-2 = \sqrt{54}-2\ ,\ a = C = t^2 = 58-2\cdot 2\sqrt{54} = 58-\sqrt{864}\ ,$$

$$b = {}^1\!/_4 C+17\,{}^1\!/_2 = 14\,{}^1\!/_2-\sqrt{54}+17\,{}^1\!/_2 = 32-\sqrt{54}\ .$$

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[11] ***(Fol 39^v)*** Quando le cose sonno oguali ali censi et al numero, se vole partire nelli censi, et poi dimezzare le cose et multiprichare per se medesimo et cavare el numero, et la radice de quello che romane, et poi el dimezzamento dele cose vale la cosa. Overo el dimezzamento dele chose meno la radice de quello che remane.

[11]*(Fol 39^v)* When the things are equal to the *censi* and to the number, one shall divide by the *censi*, and then halve the things and multiply by itself and remove the number, and the root of that which remains and then the halving of the things is the thing. Or indeed the halving of the things less the root of that which remains.

Mathematical commentary: $\beta t = \alpha C+\gamma \Rightarrow$

$$t = \sqrt{\left(\frac{\beta}{\alpha}:2\right)^2-\frac{\gamma}{\alpha}}+\left(\frac{\beta}{\alpha}:2\right) \quad\text{or}\quad t = \left(\frac{\beta}{\alpha}:2\right)-\sqrt{\left(\frac{\beta}{\alpha}:2\right)^2-\frac{\gamma}{\alpha}}\ .$$

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[12] Asemplo ala dicta regola. Et vo’ dire chosì: fammi de 10 dui parti, che multipricata la magiore contra la minore faccia 20. Adimando quanto serà ciascheuna parte. Fa chosì: poni la minore parte fosse una chosa. Dunque la magiore serà rimanente infino in 10, che serà 10 meno una chosa. Appresso si vole multiprichare la minore, che è una cosa, via la magiore, che è 10 meno una cosa. Et diciamo che vole fare 20. Et però multipricha una cosa via 10 meno una cosa. Fa 10 cose meno uno censo, la quale multiprichatione è oguale a 20. Ristora ciascheuna parte, cioè de aggiongere uno censo a ciascheuna parte, et arai che 10 cose sonno oguali a uno censo et 20 numeri. Arrecha a uno censo, et poi dimezza le cose, ve ne viene 5. Multipricha per se medesimo, fa 25. Cavane el numero, che è 20, remane 5, del quale piglia la sua radice, la quale è manifesta che non l'à apponto. Adunque vale la cosa 5, cioè el dimezzamento meno radice de 5. Et noi ponemo che la parte, cioè la minore, fosse una chosa. Adunque è 5 meno radice de 5. Et la seconda è rimanente infino in 10, che è 5 et più radice de 5. Et sta bene.

[12] Example of the said rule. And I shall say thus: Make two parts of 10 for me, so that when the larger is multiplied against the lesser, it shall make 20. I ask how much will be each part. Do thus: posit that the lesser part be a thing. Then the larger will be the remainder until 10, which will be 10 less a thing. Next one shall multiply the lesser, which is a thing, by the larger, which is 10 less a thing. And we shall say that it will make 20. And therefore multiply a thing times 10 less a thing. It makes 10 things less one *censo*, which multiplication is equal to 20. Restore each part, that is, to adjoin one *censo* to each part, and you will get that 10 things are equal to one *censo* and 20 in numbers. Bring it to one *censo*, and then halve the things, from which results 5. Multiply by itself, it makes 25. Remove from it the number, which is 20, 5 remains, of which take the root, which it is manifest that it does not have precisely. Then the thing is 5, that is, the halving less root of five. And we posited that the part, that is, the lesser, was a thing. Then it is 5 less root of 5. And the second is the remainder until 10, which is 5 and added root of 5. And it goes well.

Mathematical commentary: The problem can be expressed:

$$10 = a+b \ , \ \ a \cdot b = 20 \ .$$

Positing $a := t$ ($a<b$) we have $b = 10–t$ and thus

$$a \cdot b = t \cdot (10–t) = 10t–C = 20 \ .$$

«Restoring» we get

$$10t = C+20$$

whence, according to the rule,

$$a = t = 5 - \sqrt{5 \cdot 5 - 20} = 5–\sqrt{5} \ , \ \ b = 10–t = 5+\sqrt{5} \ .$$

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[13] Uno fa doi viaggi, et al primo viagio guadagna 12. Et al secondo viagio guadagna a quella medesema ragione che fece nel primo. Et quando che compiuti li soi viaggi et egli se trovò tra guadagniati et capitale 54. Vo' sapere con quanti se mosse. Poni che se movesse con una chosa, et nel primo viaggio guadagnio 12. Dunque compiuto el primo viaggio si trovò una cosa et 12. Adunque manifestamente vedi che de ogni una cosa nel primo viaggio fa una chosa e 12. Quanto serrà a quella medesema ragione nel secondo viaggio? Convienti multiprichare una cosa et 12 via ***(Fol 40r)*** una cosa et 12, che fa uno censo et 24 cose e 144 numeri, li quali sicondo che dice la regola si vole partire in una cosa, et dè ne venire 54. Et però multipricha 54 via una cosa. Fa 54 cose, le quali se oguagliano a uno censo et 24 cose e 144 numeri. Ristora ciascheuna parte, cioè de cavare 24 cose de ciascheuna parte. Et arai che 30 cose sonno oguali a uno censo et 144 numeri. Parti in uno censo, vene quello medesemo. Dimezza le cose, remanghono 15. Multipricha per se medesemo, fanno 225. Traine li numeri, che sonno 144. Resta 81. Trova la sua radice, che è 9. Trailo del dimezzamento dele cose, cioè de 15. Resta 6, et cotanto vale la chosa. Et noi dicemmo che se movesse con una chosa. Dunque vedi manifestamente che se mosse con 6. Et se la voi provare, fa così: tu di' che nel primo viaggio guadagnio 12 et con 6 se mosse a 18. Sì che nel primo viaggio se trovò 18. E peró di' così: se de 6 io fo 18, que farò de 18 a quella medesema ragione? Multipricha 18 via 18. Fa 324. Parti in 6, che ne vene 54, et sta bene. Et così se fanno le simili ragioni.

Ancora si poterebbe dire che si movesse colla radice de rimanente et più el dimezzamento dele cose, cioè cola radice de 81, che è 9. Pollo sopra a 15. Fa 24. Et cossì sta bene nell'uno modo como nell'altro. Et eccho la prova: noi abbiamo facta all'altro modo che se movesse con 6. Et abbiamo facto ragione che, compiuti i viaggi, si trovò 54 chomo noi diciamo. Ora faciamo ragione che se movesse con 24, et diciamo che nel primo viaggio guadagnò 12. Sì che se trovò 36. Ora di' chosì: se con 24 io fo 36, que farrò coy 36? Multiplica 36 via 36, fa 1296, et parti in 24, che ne vene 54, et sta bene. Sì che tu vedi che all'uno modo et all'altro sta bene. Et però quella così facta regola è molto da lodare, che ce dà doi responsioni et così sta bene all'una come all'altro. Ma abbi a mente che tucte le ragioni che reduchono a questa regola non si possono ***(fol 40v)*** respondere per doi responsioni se non ad certe; et tali sonno che te conviene pigliare l'una responsione, et tale l'altra. Cioè a dire che a tali ragioni te converà rispondere che vaglia la cosa el dimezzamento dele cose meno la radice de rimanente; et a tale te converrà dire la radice de remanente e più el dimezzamento dele cose. Onde ogni volta che te venisse questo [chiesto] co'tale raoguaglamento,[22] trova in prima l'una responsione. Et se non te venisse vera, de certo si piglia l'altra senza dubio. Et averai la vera responsione. Et abi a mente questa regola. In bona verità vorrebbe una grande despositione; ma non mi distendo troppo però che me pare stendere et scrivere in vile cosa; ma questo baste qui et in più dire sopra ciò non mi vo' stendere.

[13] Somebody makes two voyages, and in the first voyage he gains 12. And in the second voyage he gains at that same rate as he did in the first. And when his voyages were completed, he found to have 54, gains and capital together. I want to know with how much he set out. Posit that he set out with one thing, and in the first voyage he gained 12. Then, when the first voyage was completed, he found to have one thing and 12. Then you see manifestly that from each one thing in the first voyage he makes a thing and 12. How much will it be at that same rate in the second voyage? You ought to multiply a thing and 12 times ***(Fol 40^r^)*** a thing and 12, which makes one *censo* and 24 things and 144 in numbers, which, according to what the rule says, one shall divide by a thing, and 54 shall result from it. And therefore multiply 54 times a thing. It makes 54 things, which equal one *censo* and 24 things and 144 in numbers. Restore each part, that is, to remove 24 things from each part. And you will get that 30 things are equal to a *censo* and 144 in numbers. Divide by one *censo*, the very same results. Halve the things, 15 remain. Multiply by itself, they make 225. Detract from it the numbers, which are 144. 81 remains. Find its root, which is 9. Detract it from the halving of the things, that is, from 15. 6 remains, and as much is the thing. And we said that he set out with one thing. Then you see manifestly that he set out with 6. And if you want to verify it, do thus: you say that in the first voyage he gained 12, and with 6 he came to 18. So that in the first voyage he found to have 18. And therefore say thus: if from 6 I make 18, what will I make from 18 at that same rate? Multiply 18 times 18. It makes 324. Divide by 6, and 54 results from it, and it goes well. And thus the similar computations are made.

Again one could say that he set out with the root of the remainder and added the halving of the things, that is, with the root of 81, which is 9. Put it above 15. It makes 24. And so it goes well in one way as well as the other. And here is the verification: we have made in the other way that he set out with 6. And we have computed that, when the voyages were completed, he found to have 54, as we say. Now we compute that he set out with 24, and we say that in the first voyage he gained 12. So that he found to have 36. Now do thus: if with 24 I make 36, what will I make with the 36? Multiply 36 times 36, it makes 1296, and divide by 24, and 54 results from it, and it goes well. So that you see that it goes well in one way as well as the other. And therefore the rule so constituted is much to be praised, which gives us two answers which go well, one as well as the other. But keep in mind that not all computations that lead back to this rule can ***(fol 40^v^)*** be answered with two answers, but only some of them; and there are some for which you ought to take one answer, and some, the other. That is to say that to some computations you ought to answer that the thing is the halving of

[22] The appearance of a term for "equation" is noteworthy.

the things less the root of the remainder; and to some you ought to say the root of the remainder and added the halving of the things. Hence, every time that you are asked with such an equation, find first one answer. And if it does not turn out true for you, certainly the other is taken without doubt. And you will have the true answer. And keep in mind this rule. Verily, a vast exposition would be needed; but I will not enlarge too much, because I seem to expand and write(?) about a base thing; but this should be enough here, and I will not enlarge more upon it.

Mathematical commentary: If a is the initial capital, and p the rate of gain, the problem is

$$p\cdot a = 12\ ,\ \ a+p\cdot a+p\cdot(a+p\cdot a) = 54\ .$$

We posit $a:=t$, and get $a+p\cdot a = t+12$, whence

$$(t+12)\cdot(t+12)\,/\,t = (C+24t+144)\,/\,t = 54$$

whence

$$C+24t+144 = 54t \quad \text{or} \quad 30t = C+144\ .$$

According to the rule we therefore get an initial capital

$$a = 15-\sqrt{15\cdot 15-144} = 15-\sqrt{81} = 6\ ,$$

or, alternatively,

$$a = \sqrt{15\cdot 15-144}+15 = \sqrt{81}+15 = 24\ .$$

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[14] Pongoti assemplo a quello che abbiamo dicto denanzi, et dichò chosì: fami de 10 dui parti, che multiplicata l'una contra l'altra et sopra la dicta multiplichatione giontovi la differentia che à dall'una parte all'altra faccia 22. Adimando, quanto serrà ciascheuna parte? Fa chosì: poni che l'una parte fusse una cosa. Dunque l'altra parte serrà lo rimanente infino in 10, che serà 10 meno una cosa. Appresso multipricha l'una contra all'altra, cioè una cosa via 10 meno una cosa, che fa 10 cose meno uno censo. Appresso sopra a questa multipricatione poni la differenza che è da una cosa a 10 meno una cosa, che è 10 meno II cose, le quali differenze se vole giungere a 10 cose meno uno censo, et arai che fanno 10 numeri e otto cose meno uno censo, le quali se aggiongono a 22. Ristora ciascheuna parte, cioè de cavare 10 numeri de ciascheuna parte. Et arai che 8 cose meno uno censo sonno oguali a 12 numeri. Dà uno censo a ogni parte,[23] et arai che 8 cose sonno oguali a 12 numeri et uno censo. ***(Fol 41ʳ)*** Parti nelli censi, vene quello medesimo. Dimezza le cose, sonno 4. Multipricha per se medesimo, sonno 16. Cavane li numeri, che sonno 12, remane 4. Piglia la sua radice

[23] NB: The word «ristorare» is not used.

et più el dimezzamento dele cose. Et cotanto vale la cosa. La radice de 4 è 2. Et più el dimezzamento dele cose, che sonno 4, et 2, ày 6, et cotanto vale la chosa. Et noi dicemo che l'una parte fosse una cosa. Dunque vene ad essere 6. Et la seconda parte l'avanzo infino in 10, che è 4. Provala, et multipricha 4 via 6, fa 24. Giongi suso la differenza che è dall'una all'altra, che è 2, ài 26. Et noi vogliamo 22. Sì che vedi manifestamente che non sta bene. Però che in questa ragione la cosa non vale la radice de quello che remane et più el dimezzamente dele cose. Adunque abiamo provata questa, et non ce vene bene de certo. L'altra provamo e de certo verrà bene, cioè de pigliare el dimezzamento dele cose meno la radice de rimanente. El dimezzamento dele cose è 4. La radice de rimanente è 2. Però che como tu sai ce remase 4, et la sua radice è 2, cava 2 de 4, remane 2. Et cotanto vale la cosa. Et l'altra parte serrà rimanente infino in 10, che è 8, et sta bene. Et provala: multiplicha 2 via 8, fa 16. Poni suso la differenza ch'è da 2 a 8, che è[24] 6, a 22. Et sta bene. Et così se fanno le simiglianti ragioni.

[14] I propose to you an example of that which we have said before, and I shall say thus: make two parts of 10 for me, so that when one is multiplied against the other and above the said multiplication is joined the difference which there is from one part to the other, it make 22. I ask, how much will be each part. Do thus: posit that one part was a thing. Then the other part will be the remainder until 10, which will be 10 less a thing. Next multiply one against the other, that is a thing times 10 less a thing, which makes 10 things less one *censo*. Next, above this multiplication put the difference which there is from a thing to 10 less a thing, which is 10 less II things, which difference one shall join to 10 things less a *censo*, and you will get that they make 10 in numbers and 8 things less one *censo*, which combine to 22. Restore each part, that is, to remove 10 in numbers from each part. And you will get that 8 things less one *censo* are equal to 12 in numbers. Give one *censo* to each part, and you will get that 8 things are equal to 12 in numbers and one *censo*. ***(Fol 41^r)*** Divide by the *censi*, the very same results. Halve the things, they are 4. Multiply by itself, they are 16. Remove from them the numbers, which are 12, 4 remains. Take its root, and added the halving of the things. And as much is the thing. The root of 4 is 2. And added the halving of the things, which are 4, and 2, you get 6, and as much is the thing. And we said that one part was a thing. Then it turns out to be 6. And the second part the excess until 10, which is 4. Verify it, and multiply 4 times 6, it makes 24. Join on top the difference which there is from one to the other, which is 2, you get 26. And we want 22. So that you see manifestly that it does not go well. Therefore in this computation the thing is not the root of that which remains and added the halving of the things. Then we have verified this one, and it certainly did not result well for us. We verify the other, and it will certainly result well, that is, to take the halving of the things less the root of the remainder. The halving of the things is 4. The root of the remainder is 2. Therefore, since 4 remained for us, as

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[24] Corrected, perhaps from «fa».

you know, and its root is 2, remove 2 from 4, 2 remain. And as much is the thing. And the other part will be the remainder until 10, which is 8, and it goes well. And verify it: multiply 2 times 8, it makes 16. Put on top the difference which there is from 2 to 8, it will be to 22. And it goes well. And thus the similar computations are made.

Mathematical commentary: The problem may be expressed

$$10 = a+b\ ,\quad a\cdot b+(b-a) = 22\ .$$

Positing $a{:=}t$ we get $b = 10-t$, whence

$$t\cdot(10-t)+(10-2t) = 10+8t-C = 22 \quad \text{or} \quad 8t = 12+C\ .$$

The rule suggests the possibility

$$a = t = \sqrt{4\cdot 4-12}+4 = 2+4 = 6\ ,\quad b = 10-t = 4$$

A proof shows that $a\cdot b+(b-a) = 26$, whence the alternative possibility has to be used,

$$a = t = 4-\sqrt{4\cdot 4-12} = 4-2 = 2\ ,\quad b = 10-t = 8\ .$$

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[15] Quando li censi sonno oguali alle cose et al numero, se vole partire nelli censi, et poi dimezzare le cose, et multiplicare per se medesmo et giongere al ***(Fol 41^{v})*** numero. Et la radice dela summa più el dimezzamento dele cose vale la cosa.

[15] When the *censi* are equal to the things and to the number, one shall divide by the *censi*, and then halve the things, and multiply by itself and join to the number. And the root of the sum plus the halving of the things is the thing.

Mathematical commentary: $\alpha C = \beta t+\gamma \;\Rightarrow\; t = \sqrt{\frac{\gamma}{\alpha}+\left(\frac{\beta}{\alpha}:2\right)^2}+\left(\frac{\beta}{\alpha}:2\right)$.

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[16] Assemplo ala dicta regola. Et vo' dire così: Uno à {~~40~~} 40 fiorini d'oro et canbiòli a venetiani. Et poi de quelli venetiani tolse 60 et recambiòli a'ffiorini d'oro a uno venetiano più per fiorino che meli cambio in prima; et quando à così cambiato et quello trovò, che tra venetiani che glie rimaserò quando ne trasse 60, et li fiorini che ebe de 60 venetiani, gionti inseme fece 100. Vo' sapere quanto valze el fiorino a venetiani. Di' così: pognamo che 'l fiorino valesse una cosa. Dunque 40 fiorini vagliono 40 cose de venetiani. Poi ne tolse 60 de quelli venetiani, et cambiòli a fiorini d'oro a uno venetiano più el fiorino. Adunque cava 60 venetiani de 40 cose de venetiani. Remangono 40 cose meno 60 venetiani. Et questi venetiani che glie sono remasti, raggionti co' fiorini che egli ebe de 60 venetiani, fanno 100. Dunque se noi traessemo 40 cose meno 60 venetiani de 100, remarracte [*sic*; for «remanente», or «remarranno»?] quello che vagliono li 60 venetiani a'ffiorini d'oro. Adunque trai 40 cose meno 60 de 100, rimane 160 meno 40 cose. Et dunque li fiorini che egli ebe de 60 venetiani forono 160 meno 40 cose. Et quando egli recambiò 60 venetiani a fiorini d'oro si cambiò a uno venetiano più el fiorino che prima. Dunque li 60 venetiani cambiò a una cosa e uno venetiano. Et noi abbiamo che 60 venetiani vagliono a fiorini d'oro 160 meno 40 chose. Dunque dobbiamo sapere se 160 meno 40 chose fiorini d'oro, a avalere el fiorino una cosa et uno venetiano, se vale 60 venetiani. Adunque multiplicha 160 meno 40 cose via una cosa et uno, fanno 120 cose meno 40 censi et più 160 numeri; ***(Fol 42^r)*** sonno oguali a 60 venetiani. Et così abiamo che 120 cose meno 40 censi et più 160 numeri sonno oguali a 60. Ristora ciascheuna parte, arai che 40 censi sonno oguali a 120 cose et 100 numeri. Parti nelli censi, arai che uno censo sia oguali a 3 chose e dui numeri et mezzo. Dimezza le chose, ɪ˙/. [*sic*, for «$ɪ^1/_2$»]. Multiplicha per se medesimo, fa 2 et $^1/_4$. Giungi sopra al numero, fa 4 et $^3/_4$, et abbiamo che la chosa vale la radice de 4 et $^3/_4$ et più el dimezzamento dele chose, che fo uno ⟨e⟩ mezzo. Et noi ponemo che'l fiorino valesse una chosa, dunque valse la radice de 4 et $^3/_4$ et più el dimezzamento dele cose, che è $ɪ^1/_2$. Et è facta.

[16] Example of the said rule. And I shall say thus: Somebody has 40 gold florins and changed them to Venetians. And then from these Venetians he withdrew 60 and changed them back into florins at one Venetian more per florin than he changed them at first for me; and when he has changed thus, he found that the Venetians which remained with him when he detracted 60, and the florins he got for the 60 Venetians, joined together made 100. I want to know how much was worth the florin in Venetians. Say thus: let us posit that the florin was worth one thing. Then 40 florins are worth 40 things of Venetians. Then he withdrew 60 of these Venetians, and changed them to florins at one Venetian more the florin. Then remove 60 Venetians from 40 things of Venetians. 40 things less 60 Venetians remain. And these Venetians which remained with him, joined with the florins which he got from 60 Venetians, make 100. Then, if we detract 40 things less 60 Venetians from 100, those will remain which the 60 Venetians are worth in gold florins. Then detract 40 things less 60 from 100, 160 less 40 things remain. And then the florins which he got from 60 Venetians were 160 less 40 things. And when he changed back 60 Venetians into gold florins they were changed at one Venetian more the florin than before. Then he changed the 60 Venetians at one thing and a Venetian. And we have that 60 Venetians are worth in gold florins 160 less 40 things. Then we shall know whether 160 less 40 things gold florins, the florin being one thing and a Venetian, is worth 60 Venetians. Then multiply 160 less 40 things times one thing and one, they make 120 things less 40 *censi* and added 160 in number; ***(Fol 42^r)*** they are equal to 60 Venetians. And thus we have that 120 things less 40 *censi* and added 160 in numbers are equal to 60. Restore each part, you will get that 40 *censi* are equal to

120 things and 100 in numbers. Divide by the *censi*, you will get that one *censo* be equal to three things and two in numbers and a half. Halve the things, $1\frac{1}{2}$. Multiply by itself, it makes 2 and $\frac{1}{4}$. Join above the number, it makes 4 and $\frac{3}{4}$, and we have that the thing is the root of 4 and $\frac{3}{4}$ and added the halving of the things, which make one ⟨and a⟩ half. And we posited that the florin was worth one thing, then it was worth the root of 4 and $\frac{3}{4}$ and added the halving of the things, which is $1\frac{1}{2}$. And it is done.

Mathematical commentary: Positing t for the original rate of exchange of florins into Venetians, the quantity of Venetians first obtained is $40t$, of which 60 are changed back into $^{60}/_{t+1}$ florins and $40t$–60 remain as they are. Then

$$\frac{60}{t+1} + (40t-60) = 100 \quad \text{or} \quad \frac{60}{t+1} = 100 + 60 - 40t = 160 - 40t \quad ,$$

whence

$$(160-40t)\cdot(t+1) = 60 \quad \text{or} \quad 120t-40C+160 = 60 \; .$$

Restoring we get

$$40C = 120t+100 \; ,$$

whence

$$C = 3t+2\,^1/_2 \; .$$

According to the rule we therefore have

$$t = \sqrt{1\tfrac{1}{2}\cdot 1\tfrac{1}{2}+2\tfrac{1}{2}} + 1\tfrac{1}{2} \quad ,$$

which is the original value of the florin in Venetians.

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Qui finischo le sey regole composte con alquanti assempri. Et incomincia l'altre regole che sequitano le sopradicte sey come vederete.[25]

Here I end the six composite rules with several examples. And begins the other rules that follow those told above, as you will see.

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[17] Quando li Censi [*sic*, for «cubi»] sonno oguali al numero, si vole partire el numero per li chubi, et la radice chubicha de quello che ne vene vale la cosa.

[17] When the *censi* [*sic*, for «cubi»] are equal to the number, one shall divide the number by the *cubi*, and the cube root of that which results from it is the thing.

[25] The passage in italics is written with red ink in the ms.

Mathematical commentary: $\alpha K = \beta \Rightarrow \sqrt[3]{\frac{\beta}{\alpha}}$.

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[18] Quando li chubi sonno oguali alle cose, si vole partire le cose per li chubi, et la radice de quello che ne vene vale la cosa.

[18] When the *cubi* are equal to the things, one shall divide the things by the *cubi*, and the root of that which results from it is the thing.

Mathematical commentary: $\alpha K = \beta t \Rightarrow \sqrt{\frac{\beta}{\alpha}}$.

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[19] ⟨Q⟩uando li chusi [*sic*] sonno oguali a li censi, si vole partire li censi per li chubi. Et quello che ne vene si è numero, et cotanto vale la cosa.

[19] When the *cubi* are equal to the *censi*, one shall divide the *censi* by the *cubi*. And that which results from it is a number, and as much is the thing.

Mathematical commentary: $\alpha K = \beta C \Rightarrow \frac{\beta}{\alpha}$.

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[20] Quando li chubi et li censi sonno oguali alle chose, se vole partire nelli chubi, et poi dimezzare li censi et multiplicare per se medesimo et giongerlo ale cose. Et la radice dela somma meno el dimezzamento de' censi vale la cosa.

[20] When the *cubi* and the *censi* are equal to the things, one shall divide by the *cubi*, and then halve the *censi* and multiply by itself and join to the things. And the root of the sum less the halving of the *censi* is the thing.

Mathematical commentary: $\alpha K+\beta C = \gamma t \Rightarrow \sqrt{\frac{\gamma}{\alpha} + (\frac{\beta}{\alpha}:2)^2} - (\frac{\beta}{\alpha}:2)$.

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[21] Quando li censi sonno oguali alli chubi et alle cose, ***(Fol 42^v)*** devi partire nelli chubi et poi dimezzare li censi et multiplicare per se medesimo et cavarne le cose, et la radice de quello ⟨che⟩ rimane più el dimezzamento deli censi vale la cosa. Overo el dimezzamento de' censi meno la radice de rimanente.

[21] When the *censi* are equal to the *cubi* and the things, ***(Fol 42^v)*** you shall divide by the *cubi* and then halve the *censi* and multiply by itself and remove from it the things, and the root of that ⟨which⟩ remains plus the halving of the *censi* is the thing. Or indeed the halving of the *censi* less the root of the remainder.

Mathematical commentary: $\beta C = \alpha K + \gamma t \Rightarrow \sqrt{(\frac{\beta}{\alpha}:2)^2 - \frac{\gamma}{\alpha}} + (\frac{\beta}{\alpha}:2)$

or $(\frac{\beta}{\alpha}:2) - \sqrt{(\frac{\beta}{\alpha}:2)^2 - \frac{\gamma}{\alpha}}$.

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[22] Quando li chubi sonno oguali alli censi et alle cose, dei partire ⟨ne⟩li chubi et poi dimezzare li censi, et multiplichare per se medesimo et agiungere alle cose, et la radice dela summa più el dimezzamento de' censi vale la cosa.

[22] When the *cubi* are equal to the *censi* and the things, you shall divide ⟨by⟩ the *cubi* and then halve the *censi*, and multiply by itself and join to the things, and the root of the sum plus the halving of the *censi* is the thing.

Mathematical commentary: $\alpha K = \beta C + \gamma t \Rightarrow \sqrt{\frac{\gamma}{\alpha} + (\frac{\beta}{\alpha}:2)^2} + (\frac{\beta}{\alpha}:2)$.

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[23] Quando li censi de censi sonno oguali al numero, se vole partire el numero nelli censi de censi. Et la radice ⟨della radice⟩ de quello che ne vene vale la cosa.

[23] When the *censi de censi* are equal to the number, one shall divide the number by the *censi de censi*. And the root ⟨of the root⟩ of that which results from it is the thing.

Mathematical commentary: $\alpha CC = \beta \Rightarrow t = \sqrt{\sqrt{\frac{\beta}{\alpha}}}$.

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[24] Quando li censi de censi sonno oguali alle cose se vole partire le cose per li censi de censi, et la radice chubicha de quello vale la cosa.

[24] When the *censi de censi* are equal to the things one shall divide the things by the *censi de censi*, and the cube root of that is the thing.

Mathematical commentary: $\alpha CC = \beta t \Rightarrow t = \sqrt[3]{\frac{\beta}{\alpha}}$.

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[25] Quando li censi de censi sonno oguali a censi, se vole partire li censi per li censi de censi, et la radice de quello che ne vene vale la chosa.

[25] When the *censi de censi* are equal to the *censi*, one shall divide the *censi* by the *censi de censi*, and the root of that which results from it is the thing.

Mathematical commentary: $\alpha CC = \beta C \Rightarrow t = \sqrt{\frac{\beta}{\alpha}}$.

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[26] Quando li censi de censi sonno oguali ali chubi, se vole partire li chubi per li censi de censi. Et quello che ne vene si è numero, et cotanto vale la cosa.

[26] When the *censi de censi* are equal to the *cubi*, one shall divide the *cubi* by the *censi de censi*. And that which results is a number, and as much is the thing.

Mathematical commentary: $\alpha CC = \beta K \Rightarrow t = \frac{\beta}{\alpha}$.

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[27] Quando li censi de censi et li chubi sonno oguali ali censi, si vole partire nelli censi de censi, et poi dimezzare li chubi et multiplichare per se medesimo, et agiungere alli censi. Et la radice dela summa meno el dimezzamento de' chubi vale la cosa.

[27] When the *censi de censi* and the *cubi* are equal to the *censi*, one shall divide by the *censi de censi*, and then halve the *cubi* and multiply by itself, and adjoin to the *censi*. And the root of the sum less the halving of the *cubi* is the thing.

Mathematical commentary: $\alpha CC + \beta K = \gamma C \Rightarrow \sqrt{\frac{\gamma}{\alpha} + (\frac{\beta}{\alpha} : 2)^2} - (\frac{\beta}{\alpha} : 2)$.

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[28] Quando li chubi sonno oguali alli censi de censi et {d}a censi, si vole partire nelli censi de censi, et poi dimezzare li chubi, et multiplichare per se medesimo, et cavarne li censi et la radice dela summa [*sic*] et el dimezzamento de’ chubi vale la chosa. Overo el dimezzamento de’ chubi meno la radice de quello che remane.

[28] When the *cubi* are equal to the *censi de censi* and to *censi*, one shall divide by the *censi de censi*, and then halve the *cubi*, and multiply by itself, and remove from it the *censi*, and the root of the sum [*sic*] and the halving of the *cubi* is the thing. Or indeed the halving of the *cubi* less the root of that which remains.

Mathematical commentary: $\beta K = \alpha CC+\gamma C \Rightarrow$

$$t = \sqrt{(\frac{\beta}{\alpha}:2)^2 - \frac{\gamma}{\alpha}} + (\frac{\beta}{\alpha}:2) \quad \text{or} \quad t = (\frac{\beta}{\alpha}:2) - \sqrt{(\frac{\beta}{\alpha}:2)^2 - \frac{\gamma}{\alpha}}.$$

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[29] ***(Fol 43^r)*** Quando li censi de censi sonno oguali a chubi et a censi, vole partire nelli censi de censi, et poi dimezzare li chubi, et multiplichare per se medesimo, et giungere alli censi. Et la radice dela summa più el dimezzamento de’ chubi vale la cosa.

[29] When the *censi de censi* are equal to *cubi* and to *censi*, one shall divide by the *censi de censi*, and then halve the *cubi*, and multiply by itself, and join to the *censi*. And the root of the sum plus the halving of the *cubi* is the thing.

Mathematical commentary: $\alpha CC = \beta K+\gamma C \Rightarrow t = \sqrt{\frac{\gamma}{\alpha} + (\frac{\beta}{\alpha}:2)^2} + (\frac{\beta}{\alpha}:2).$

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[30] Quando li censi de censi et li censi sonno oguali al numero, se vole partire nelli censi de censi, et poi dimezzare li censi ⟨et multiplichare per se medesimo⟩ et agiungere al numero. Et la radice dela radice dela summa et meno el dimezzamento de’ censi vale la cosa.

[30] When the *censi de censi* and the *censi* are equal to the number, one shall divide by the *censi de censi*, and then halve the *censi* ⟨and multiply by itself⟩ and adjoin to the number. And the root of the root of the sum and less the halving of the *censi* is the thing.

Mathematical commentary: $\alpha CC + \beta C = \gamma \;\Rightarrow\; t = \sqrt{\sqrt{\frac{\gamma}{\alpha} + (\frac{\beta}{\alpha}:2)^2} - (\frac{\beta}{\alpha}:2)}$.

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Qui finischono le XV[26] regole sopradicte senza niuna dispositione, le qual' cose como io t'ò dicto se reduchono alle sey regole de prima.

Here end the XV rules mentioned above without any exposition, which as I have said to you lead back to the six rules from before.

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[31] Uno homo à 100 staia de grano che vale soldi 20 lo staio, et grano che vale soldi 12 lo staio. Ora vene per caso che costui vole mettere, de quello che vale soldi 12 lo staio, sopra a quello che vale soldi 20 lo staio, tanto che così mescolato vagha soldi 18 lo staio. Vo' sapere quanto ve ne mettarà. Fa così: pogniamo che li ponggi cossì ordinati, et di' così: da soldi 12 lo staio infino a quello da soldi 18 sia soldi VI. Et poni 6 sopra a quello da 20 lo staio. Et poi di' così: da soldi 20 infino in soldi 18 sia 2. Et poni 2 sopra a quello de soldi 12 lo staio. Ora di' così: quando tolgho staia 6 de quello che vale lo staio soldi 20, sì tolgo staia 2 de quello chavale soldi 12. Vo' sapere, quando io torrò staia 100 de quello de soldi 20 lo staio, quanto torrò de quello de soldi 12? Et però, 100 via 2 staia de soldi 12 lo staio fa staia 200. Et parti in 6 che ne vene staia 33 e $^1/_3$ staio.[27] Sì che se tu metterai staia 33 e $^1/_3$ staio de quello che vale soldi 12 lo staio, sopra ad staia 100 de soldi 20 lo staio, arai in tucto staia 133 e $^1/_3$ de grano de soldi 20 et de soldi 12 lo staio. Ora la prova se sta bene: Tu di' che volini potere dare per soldi 18 lo staio così miscolato. Sappi prima che vagliono staia 100 de soldi 20 lo staio, che vale

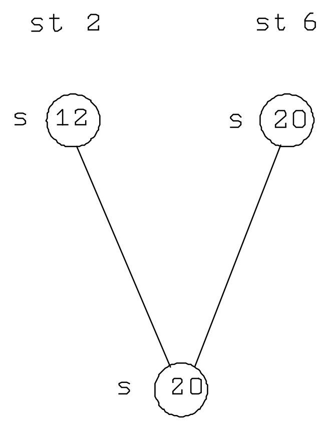

Diagram in the margin. The «20» in bottom should evidently be «18».

[26] In fact, the manuscript only contains 14 rules. The omission will have regarded the analogue of either the second or the third mixed second-degree case, respectively

Censi oguali a censi de censi et numero

and

Censi de censi oguali a censi et numero.

[27] The margin summarizes "Staia 33 $^1/_3$ staio".

libre 100. Ora sappi quello che vale staia 33 e $^1/_3$ de soldi 12 lo staio, che vale libre 20. Poni sopra a ***(Fol 43ᵛ)*** 100 libre et ài 120 libre. Et cotanto vale così miscolato le 133 staio e $^1/_3$ per soldi 18 lo staio, che vagliono appunto libre 120 et sta bene et è facta. Et così se fanno tucte le simili ragioni.

[31] A man has 100 *staia* of grain that is worth 20 *soldi* the *staio*, and grain that is worth 12 *soldi* the *staio*. Now it happens accidentally that he wants to put, of that which is worth 12 *soldi* the *staio*, so much above that which is worth 20 *soldi* the *staio* that, thus blended, it be worth 18 *soldi* the *staio*. I want to know how much of it he will put to it. Do thus: let us posit that you posit them thus ordered, and say thus: from 12 *soldi* the *staio* until that of 18 *soldi* let there be VI *soldi*. And posit 6 above that of 20 the *staio*. And then say thus: from 20 *soldi* until 18 *soldi* let there be 2. And posit 2 above that of 12 *soldi* the *staio*. Now say thus: When I withdrew 6 *staia* from that which is worth 20 *soldi* the *staio*, 2 *staia* are withdrawn from that which is worth 12 *soldi*. I want to know, when I shall withdraw 100 *staia* from that of 20 *soldi* the *staio*, how much shall I withdraw from that of 12 *soldi*? And therefore, 100 times 2 *staia* of 12 *soldi* makes 200 *staia*. And divide by 6, and 33 *staia* and $\frac{1}{3}$ *staio* result from it. So that if you will put 33 *staia* and $\frac{1}{3}$ *staio* of that which is worth 12 *soldi* the *staio*, above 100 *staia* of 20 *soldi* the *staio*, you will get in total 133 *staia* and $\frac{1}{3}$ of grain of 20 *soldi* and of 12 *soldi* the *staio*. Now the verification whether it goes well: You say that they will be able to give (i.e., sell) for 18 *soldi* the *staio* thus blended. Know first what are worth 100 *staio* of 20 *soldi* the *staio*, which is worth 100 *libre*. Now know that which is worth 33 *staia* and $\frac{1}{3}$ of 12 *soldi* the *staio*, which is worth 20 *libre*. Put it above ***(Fol 43ᵛ)*** 100 *libre* and you will get 120 *libre*. And as much is worth, thus blended, the 133 *staio* and $\frac{1}{3}$ of 18 *soldi* the *staio*, which are worth precisely 120 *libre*, and it goes well, and it is done. And thus all the similar computations are made.

Mathematical commentary: No algebra is used. Instead, if *a* is the quantity of grain of 12 *soldi* the *staio*, the solution makes use of the alligation principle

$$\frac{a}{100} = \frac{20-18}{18-12} .$$

The diagram recurs thrice on fols 48ᵛ–50ʳ in problems concerned with alligation of bullion.

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[32] Uno sta a uno fondacho 3 anni, et à de salario tra'l primo anno e'l terzo 20 fiorini. El secondo anno à 8 fiorini. Vo' sapere que glie venne el primo anno et que el terzo precisamente [?], ogni uno per se solo. Fa così, et questo te sia sempre a mente, che tanto vole fare multiplichato el secondo anno per se medesimo quanto el primo nel terzo. Et fa così: multiplicha el secondo per se medesimo che di' che ebe 8 fiorini. Multipricha 8 via 8, fa 64 fiorini. Ora te convene fare de 20 fiorini, che tu di' che ebbe tra'l primo e'l terzo anno, tra 2 parti che moltipricha⟨ta⟩ l'una contra l'altra faccia 64 fiorini. Et farrai così, cioè che sempre dimezze quello che à nelli 2 anni. Cioè, dimezza 20, venne 10. Moltipricha l'uno contra all'altro, fa 100. Cavane la multiprichatione facta del secondo anno che è 64, resta 36. Et de questo trova la sua radice, et dirrai che l'una parte serà 10, cioè el primo anno [*sic*, this order] meno radice de 36. Et l'altra parte, cioè el secondo anno, serà 8 fiorini. Et la terza serà da 10 meno radice de 36 infino in 20 fiorini, che sonno fiorini 10 et più radice de 36. Et se la voli provare, fa così et di': el primo anno à 10 fiorini meno radice de 36 che è 6. Tray 6 de 10, resta 4 fiorini. Et 4 fiorini ebbe el primo anno. Et el secondo ebe 8 fiorini. Et el terzo ebbe fiorini 10 et più radice de 36, che è 6. Ora poni 6 fiorini sopra a 10 fiorini, arai 16 fiorini. Et tanto ebe el terzo anno. Et sta bene. Et tanto fa multiprichato el primo contra al terzo quanto el secondo per se medesimo. Et tal parte è el secondo del terzo quale el primo del secondo. Et è fatta.

[32] Somebody is in a warehouse[28] 3 years, and in the first and third year together he

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[28] I have found three other problems where the salary of the manager of a *fondaco* is supposed to increase in geometric progression. The first is Paolo dell'abbaco's fourteenth-century *Trattato d'aritmetica* [ed. Arrighi 1964: 149] (actually an extract from Paolo, see the incipit, ed. [Van Egmond 1980: 114]), where the increase of the salary is still taken for granted (whence we may conclude that the problem was still a familiar standard problem). The second is Benedetto da Firenze's selection from maestro Biagio's collection of algebra problems [ed. Pieraccini 1983: 89–91], dated before 1340. Benedetto, writing in 1463, explains the presupposed increase meticulously, thereby implying that the type has disappeared since Biagio's times – indeed, after Paolo and Biagio, problems on *fattori* and *fondachi* have a different (and more realistic) mathematical structure – cf. [Tropfke / Vogel et al 1980: 559*f*]. The third, in fact, does not deal with a *fondaco* and its manager but with the wages of a servant: Filippo Calandri's late-fifteenth century problem collection [ed. Santini 1982: 32*f*]. It has the structure of Jacopo's [34], and tells the type of increase.

Paolo's simple problem (though restricted to three years) has the structure of Jacopo's [33]. Benedetto's coincides with Jacopo's [35] apart from a factor 2, but it is solved by means of algebra. Filippo's solution coincides in detail with Jacopo's – and so do many of his formulations. It is noteworthy that Filippo's no 44 is a three-participant analogue of Jacopo's [10], involving a square root in the data.

A pure-number version of Jacopo's [35] (with sums 26 and 39, which yields a solution in integer numbers) is found in another part of Benedetto's compilation, namely in his selection from Antonio de' Mazzinghi's *Fioretti* [ed. Arrighi 1967: 5]. Even in this case the solution is algebraic, though different from Maestro Biagio's. Antonio is regarded the most brilliant disciple of Paolo dell'Abbaco.

gets a salary of 20 florins. The second year he gets 8 florins. I want to know precisely what he received the first year and the third year, each one by itself. Do thus, and let this always be in your mind, that the second year multiplied by itself will make as much as the first in the third. And do thus: multiply the second by itself, in which you say that he got 8 florins. Multiply 8 times 8, it makes 64 florins. Now you ought to make of 20 florins, which you say he got in the first and third year together, two parts which, when multiplied one against the other, make 64 florins. And you will do thus, that is that you always halve that which he got in the two years. That is, halve 20, 10 result. Multiply the one against the other, it makes 100. Remove from it the multiplication made from the second year which is 64, 36 remains. And of this find its root, and you will say that one part, that is, the first year,[29] will be 10 less root of 36. And the other part, that is, the second year, will be 8 florins. And the third will be from 10 less root of 36 until 20 florins, which are 10 florins and added root of 36. And if you want to verify, do thus and say: the first year he gets 10 florins less root of 36, which is 6. Detract 6 from 10, 4 florins remains. And 4 florins he got the first year. And the second year he got 8 florins. And the third he had 10 florins and added root of 36, which is 6. Now put 6 florins above 10 florins, you will get 16 florins. And so much did he get the third year. And it goes well. And the first multiplied against the third makes as much as the second by itself. And such a part is the second of the third as the first of the second. And it is done.

Mathematical commentary: If the salaries in the three years are a, b and d, we know that $a+d = 20$, and $b = 8$. Moreover, the three are presupposed to be in continued proportion, whence $ad = b^2 = 64$. As in [12] we thus have two numbers, whose sum and product we know. However, the problem is not solved by means of *censo* and thing, nor according to an algorithm derived from the solution of [12]; instead, the two numbers are found as

$$a = \frac{a+d}{2} - \sqrt{\left(\frac{a+d}{2}\right)^2 - ad} \quad \text{and} \quad b = \frac{a+d}{2} + \sqrt{\left(\frac{a+d}{2}\right)^2 - ad} .$$

This corresponds to the procedure used in Abū Bakr's *Liber mensurationum*, No. 25 [ed. Busard 1968: 25] and to Diophantos's *Arithmetica* I.27 [ed., trans.

It should perhaps be mentioned that proposition III.5 of Jordanus of Nemore's *De numeris datis* [ed. Hughes 1981] is the pure-number version of Jacopo's [32]. Everything, however, speaks against a connection: Jordanus's solution is different (it goes via sum and difference, not half-sum and half-difference). Moreover, the analogues of [34] and [35] are absent from Jordanus's treatise, even though his III.18 and III.19 are their analogues for four numbers in non-continued proportion; if Jordanus had known Jacopo's problems, he would certainly have included them in his rather disparate collection of problems about numbers in proportion.

[29] Order of phrases corrected.

Tannery 1893: I, 60–62].

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[33] ***(Fol 44^r^)*** Uno sta a uno fondicho 4 anni, et el primo anno ebe 15 fiorini d'oro; el quarto ebe 60 fiorini. Vo' sapere quanto ebe el secondo anno e'l terzo a quella medesima ragione. Fa così: che tu partequello che egli ebbe el quarto anno in quello che ebbe el primo anno. Et dirai che quello che ne vene sia radice chubicha. Ora ài a partire 60 fiorini in 15, che ne vene 4 fiorini. Et questo 4 si è radice chubicha. Et sempre piglia el partitore et arrechalo a radice, cioè arrecha 15 a radice, et di' chosì: Multipricha 15 via 15, fa 225 [corrected from «125»]. Ora multipricha 15 via 225, che fa 3375. Ora multipricha la radice chubica, cioè 4, che è radice chubicha, contra ala radice chubicha {contra ala radice chubicha} de 3375 che fa radice chubicha de 13500. Et cotanto ebbe el secondo anno. Ora facciamo per lo terzo anno et multipricha 4, che è dicto de sopra, contra a radice chubicha de 13500, che fa radice chubicha 54000, et cotanto ebbe el terzo anno ad quella medesema ragione che ebbe el primo e'l quarto anno. Sì che noi dirremo che costui avesse el primo anno fiorini 15. El secondo anno ebbe radice chubica de 13500 fiorini d'oro. El terzo anno ebbe radice chubicha de fiorini 54000, et el quarto anno ebe fiorini 60 d'oro. Et sta bene.

[33] Somebody is in a warehouse 4 years, and in the first year he got 15 gold florins; the fourth he got 60 florins. I want to know how much he got the second year and the third at that same rate. Do thus: that you divide that which he got in the fourth year by that which he got in the first year. And you will say that what resulted from it is a cube root. Now you have to divide 60 florins by 15, from which results 4 florins. And this 4 is a cube root. And always take the divisor and bring it to root, that is, bring 15 to root, and say thus: multiply 15 times 15, it makes 225. Now multiply 15 times 225, which makes 3375. Now multiply the cube root, that is 4, which is cube root, against the cube root of 3375, which makes the cube root of 13500. And as much did he get the second year. Now let us do for the third year and multiply 4, which is said above, against the cube root of 13500, which makes the cube root of 54000, and as much did he get the third year at that same rate as he had the first and the fourth year. So that we will say that he got the first year, 15 florins. The second year he got cube root of 13500 gold florins. The third year he got cube root of 54000, and the fourth year 60 gold florins. And it goes well.

Mathematical commentary: If the respective salaries are a, b, d and e, we know that $a = 15$, $e = 60$, and that the salaries are in continued proportion. The solution is given as

$$b = \sqrt[3]{\frac{d}{a} \cdot a^3} \quad , \quad d = \sqrt[3]{(\frac{d}{a})^2 \cdot a^3} \quad .$$

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[34] Uno sta a uno fondicho 4 anni. Et tra'l primo anno e'l quarto ebe 90 fiorini d'oro. Et tra'l secondo anno e'l terzo ebbe 60 fiorini d'oro. Vo' sapere que gli venne ogni uno per se solo. Et siano in propositione et sia tal parte el primo del secondo come el secondo del terzo, et come el terzo del quarto; et sempre te stia a mente questo, che tanto fa a multiprichare el primo anno nel quarto quanto el secondo anno nel terzo. Et tanto fa a partire el quarto anno nel secondo quanto el terzo anno nel primo. Ora fa così, che sempre tu arreche quello che egli à tra'l secondo e'l terzo anno a radice chubicha. Et poi multipricha quello ***(fol 44^v)*** che egli à tra'l secondo e'l terzo anno per 3. Et sopra aquello giongi quello che gl'à tra'l primo e'l quarto anno. Et questo è el partitore. Et ài a partire la radice chubicha sopradicta. Et per che tu intende meglio, fa così: multipricha 60 via 60, fa 3600. Et 60 via 3600 fa 216000, et ài a partire in quello che fa 3 via 60 giontovi suso 90, che fa 270. Et questo è el partitore. Parti 216000 in 270, che ne vene 800. Et tanto fa multiprichato el primo anno nel quarto; et multiprichato el secondo nel terzo fa ancho 800. Sì che te convene fare de 90 doi parti, che multiprichata l'una contra l'altra faccia 800. E però fa così: dimezza 90, venne 45. Moltiprichalo per se medesimo, fa 2025. Cavane 800, resta 1225. Et dirai che l'una parte, cioè el primo anno, avesse fiorini 45 meno radice de 1225 fiorini. Et el quarto anno lo resto infine in 90 fiorini che è fiorini 45 et più radice de 1225 fiorini. Et afacto pe'l primo e'l quarto anno. Et per che tu intende meglio questo numero, cioè 1225, la sua radice si è 35, però che fa 35 via 35 1225. Sì che el primo anno di' che ebbe fiorini 45 meno 35, resta 10 fiorini. Et fiorini 10 ebe el primo anno. El quarto anno ebe fiorini 45 di et [*sic*, this order] più radice de 1225, che è fiorini 35. Poni sopra a 45, fa 80. Et fiorini 80 ebbe el quarto anno. Ora facciamo per lo secondo et terzo anno et fa in simile modo: che tu faccia de 60ta 2 parti che multiprichata l'una contra all'altra faccia 800. Et però fa così: dimezza 60, venne 30. Multipricha per se medesimo, fa 900. Traine 800, resta 100. Et dirai che'l secondo anno avesse fiorini 30 meno radice de 100. Et el terzo anno el resto infino {«~~el secondo~~» crossed out} in 60, che è 30 et più radice de 100. Et la radice de 100 si è 10, sì como tu sai, 10 via 10 fa 100. Et però perché tu di' che'l secondo anno à fiorini 30 meno {meno} radice de 100 fiorini, che sonno fiorini 10, trai 10 de 30, resta 20. Et fiorini 20 ebbe el secondo anno. Et el terzo ebbe fiorini 30 et più radice de fiorini 100 che è 10. Poni 10 sopra a 30, fa 40, et 40 fiorini ebe el terzo anno. Et è facta, et bene vedi chiaro che ciascheuno de questi numeri sonno in propositione. Et tal parte ***(fol 45^r)*** è el primo del secondo quale el secondo del terzo et quale el terzo del quarto: ciascheuno è la mità. Et anchora vedi chiaro che tanto fa multiplicato el primo contra al quarto, che fa tanto quanto multiplichato el secondo contra al terzo. Et tante ne vene a partire el quarto nel secondo quanto vene a partire el terzo nel primo. Sì che vedi chiaro che la allegatione sta bene. Et è facta apponto. Et così se fanno le simiglianti ragioni.

El primo	anno	ebbe como ài veduto	fiorini 10 d'oro appunto.
El secondo	anno	ebbe ————————	fiorini 20 d'oro appunto.
El terzo	anno	ebbe ————————	fiorini 40 d'oro appunto.
El quarto	anno	ebbe ————————	fiorini 80 d'oro appunto.

[34] Somebody is in a warehouse 4 years. And in the first year and the fourth year together he got 90 gold florins. And in the second year and the third year together he got 60 gold florins. I want to know what resulted for him, each one by itself. And let them be in proportion and let the first be such part of the second as the second of the third, and as the third of the fourth; and let it always stay in your mind that to multiply the first year by the fourth makes as much as the second year by the third. And it makes as much to divide the fourth year by the second as the third year by the first. Now do thus, that you always bring that which he gets the second and third year together to cube root. And then multiply that which he gets in the second and third year together by 3. And above this you join that which he gets in the first and fourth year together. And this is the divisor. And you have to divide the cube root said above. And in order that you understand better, do thus: multiply 60 times 60, it makes 3600. And 60 times 3600 makes 216000, and you have to divide by that which 3 times 60 makes when 90 is joined on top, which makes 270. And this is the divisor. Divide 216000 by 270, from which results 800. And so much makes the first year multiplied by the fourth; and the second multiplied by the third again makes 800. So that you ought to make from 90 two parts, of which one, multiplied against the other, makes 800. And therefore, do thus: halve 90, 45 result. Multiply it by itself, it makes 2025. Remove 800 from it, 1225 remains. And you will say that one part, that is, the first year, he got 45 florins less root of 1225 florins. And the fourth year the rest until 90 florins, which is 45 florins and added root of 1225 florins. And in fact[30] for the first and the fourth year. And in order that you understand better this number, that is, 1225, its root is 35 therefore that 35 times 35 makes 1225. So that the first year you say that he got 45 florins less 35, 10 florins remains. And 10 florins he got the first year. The fourth year he got 45 florins and added root of 1225, which is 35 florins. Put it above 45, it makes 80. And 80 florins he got the fourth year. Now let us do for the second and third year and do in a similar way: that

[30] Or "it is done", if we take the unambiguous "afacto" of the manuscript as a writing error for "e facto".

you make from 60 2 parts so that, when one is multiplied by the other, it makes 800. And therefore do thus: halve 60, 30 results. Multiply by itself, it makes 900. Detract from it 800, 100 remains. And you will say that the second year he had 30 florins less root of 100, And the third year the rest until 60, which is 30 and added root of 100. And the root of 100 is 10, since as you know, 10 times 10 makes 100. And therefore, since you say that the second year he gets 30 florins less root of 100 florins, which are 10 florins, detract 10 from 30, 20 remains. And 20 florins he got the second year. And the third he got 30 florins and added root of 100 florins, which is 10. Put 10 above 30, it makes 40, and 40 florins he got the third year. And it is done, and you see well clearly that all of these numbers are in proportion. And such part ***(fol 45^r^)*** is the first of the second as the second of the third, and as the third of the fourth: each is the half. And again you see clearly that the first multiplied against the fourth makes as much as the second makes when multiplied against the third. And as much results from it when the fourth is divided by the second as results when the third is divided by the first. So that you see clearly that the composition goes well. And it is done precisely. And thus the similar computations are made.

The first	year	he got, as you have seen,	precisely 10 gold florins.
The second	year	he got ————————	precisely 20 gold florins.
The third	year	he got ————————	precisely 40 gold florins.
The fourth	year	he got ————————	precisely 80 gold florins.

Mathematical commentary: If the respective salaries are a, b, d and e, we know that $a+e = 90$, $b+d = 60$, and that the numbers are in continued proportion. The solution makes use of the formula

$$a \cdot e = b \cdot d = \frac{(b+d)^3}{3(b+d)+(a+e)}$$

The formula is easily justified algebraically, if we introduced p as the ratio of subsequent salaries, whence $b = ap$, $d = ap^2$, $e = ap^3$, since then

$$\frac{(b+d)^3}{3(b+d)+(a+e)} = \frac{a^3p^3(1+p)^3}{a(3p+3p^2+1+p^3)} = a^2p^3 = a \cdot ap^3 = ap \cdot ap^2 .$$

This, however, is not likely to be a faithful rendering of the underlying reasoning.

The determination, respectively, of a and e and b and d from their sum and product follows the pattern of [32].

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[35] Uno sta a uno fundecho 4 anni. Et tra'l primo anno e'l terzo ebe fiorini 20 d'oro. Et tra'l secondo e'l quarto anno ebbe {fiorini} 30 fiorini d'oro. Vo' sapere que glie toccho el primo anno e'l secondo e'l terzo e'l quarto. Et che tal parte sia el primo del secondo quale è el terzo del quarto. Fa così, et questo abbi sempre per regola, che tu parte sempre quello che gl'à tra'l secondo et quarto anno in quello che gl'à tra'l primo e'l terzo. Et ciò che ne vene multiplichalo per se medesimo; et sopra quello che fa, sempre poni uno per regola, et quello che fa si è el partitore. Et in quello ài a partire amendori li salarii, cioè quello che egli à in questi 4 anni, ciascheuno salario de per se. Et per che tu intende meglio, fa così como di sopra abiamo dicto: che tu parti quello che gl'à tra'l secondo et quarto anno in quello che egli à tra'l primo e'l terzo. Et però fa così: parti fiorini 30 in fiorini 20, che ne vene fiorini I $^1/_2$. Multiprichalo per se medesimo, fa 2 et $^1/_4$. Ponni suso uno, como dice la regola, fa 3 e $^1/_4$. Et questo è el partitore. Ora parti 20 fiorini, che egli à tra'l primo e'l terzo anno in 3 e $^1/_4$, che ne vene fiorini 6 e $^2/_{13}$ de fiorino. Et tanto glie toccha ***(Fol 45^v)*** el primo anno. Et el terzo anno el resto infino in 20 fiorini che è fiorini 13 e $^{11}/_{13}$ de fiorino. Ora parti fiorini 30 in 3 e $^1/_4$, venne fiorini 9 e $^3/_{13}$ de fiorino. Et tanto gle toccha el secondo anno. Et el resto infino in 30 fiorini gle toccha el quarto anno, che è fiorini 20 e $^{10}/_{13}$ de fiorini. Et è facta, et vedi che sonno li salarii in propositione, che tal parte è el primo del secondo quale {erasure} el terzo del quarto. Et tal parte è el primo del secondo quale el secondo del terzo; et quale è el terzo del quarto; che ciascheuno numero è $^2/_3$ dell'altro. Et sta bene. Et così se fanno tucte le simigliante ragioni.

El primo	anno	ebbe ——	fiorini 6	e $^2/_3$ de fiorino
El secondo	anno	ebbe ——	fiorini 9	e $^3/_{13}$ de fiorino
El terzo	anno	ebbe ——	fiorini 13	e $^{11}/_{13}$ de fiorino
El quarto	anno	ebbe ——	fiorini 20	e $^{10}/_{13}$ de fiorino

[35] Somebody is in a warehouse 4 years. And in the first year and the third together he got 20 gold florins. And in the second and the fourth year he got 30 gold florins. I want to know what went to him the first year and the second and the third and the fourth. And that the first be such part of the second as the third is of the fourth. Do thus, and have this always as a rule, that you always divide that which he has in the second and fourth year together by that which he had in the first and third together. And that which results from it, multiply it by itself; and above that which it makes, always put one by rule, and that which it makes is the divisor. And by this you have to divide both salaries, that is, that which he got in these 4 years, each salary by itself. And in order that you understand better, do thus as we have said above: that you divide that which he got in the second and fourth year together by that which he got in the first and third together. And therefore do thus: divide 30 florins by 20 florins, from which results I $^1/_2$ florins. Multiply it by itself, it makes 2 and $^1/_4$. Put one on top, as the rule says, it makes 3 and $^1/_4$. And this is the divisor. Now divide 20 florins, which he gets in the first and third year together, by 3 and $^1/_4$, from which results 6 florins and $^2/_{13}$ of a florin. And so much goes to him ***(Fol 45^v)*** the first year. And the third year the rest until 20 florins, which is 13 florins and $^{11}/_{13}$ of a florin. Now divide 30 florins by 3 and $^1/_4$, 9 florins and $^3/_{13}$ of a florin results.

And so much goes to him the second year. And the rest until 30 florins goes to him the fourth year, which is 20 florins and $^{10}/_{13}$. And it is done, and you see that the salaries are in proportion, that the first is such part of the second as the third of the fourth. And such part is the first of the second as the second of the third; and as the third of the fourth; that each number is $^{2}/_{3}$ of the other. And it goes well. And thus all the similar computations are made.

The first	year	he had	6 florins and	$^{2}/_{3}$ of a florin
The second	year	he had	9 florins and	$^{3}/_{13}$ of a florin
The third	year	he had	13 florins and	$^{11}/_{13}$ of a florin
The fourth	year	he had	20 florins and	$^{10}/_{13}$ of a florin

Mathematical commentary: If the respective salaries are a, b, d and e, we know that $a+d = 20$, $b+e = 30$, and that the numbers are in continued proportion. Without being so identified, the ratio p is found as $(b+e)/(a+d)$, whence

$$a = \frac{a+d}{1+p^2}\ ,\ \ d = (a+d)-a\ ,\ \ b = \frac{b+e}{1+p^2}\ ,\ \ e = (b+e)-b\ .$$

Appendix: two non-algebraic problems

After an extensive section dealing with alligations, bullion and exchagne of money comes the following problem:

(Fol 50^v^)

[...]

[A1] Uno homo toglie una boctegha a'ppeggione, et venni a stare dentro in kalende gienaro. Ora viene un altro, acconpagnasse colui in kalende aprile. Viene un altro, accompagnase coloro kalende luglio. Viene un altro, accompagnase coloro in kalende ottobre. El primo mette in compagnia, cioè mise en la boctegha el primo dì che la tolze a pegione libre 100. El secondo mise el dì che se accompagniò con loro libre 200. El terzo mise libre 300. El quarto mise libre 400. Et così stanno tucti e quattro inseme infino in kalende gienaro. Et in capo dell'anno elli vegono loro conto. Et trovasi guadagnato libre 100. Adomandoti como ***(fol. 51^r^)*** sa⟨rà⟩ a partire questo guadagnio, et quello toccharà per uno. Devi fare così: Merita ciascheuno li soi denari per lo tempo che egli è stato nela compangia, a 2 denari per libra el mese. Et diciamo così: El primo è stato in compagnia uno anno, et misse libre 100, che dè avere de merito libre 10. Et colui che mise 200, cioè el secondo, è stato in compagnia mesi 9, che dè avere de merito libre 15. El terzo, che mise libre 300, è stato in compagnia mesi 6; dè avere de merito libre 15. El quarto, che mise in compagnia libre 400, dè avere de merito per tre mesi libre 10. Ora di' così: E sonno 4 compagni che ànno facto compagnia inseme. Et l'uno mette in compagnia libre 10. Et l'altro libre 15. Et l'altro libre 15. Et l'altro libre 10. Et ànno guadagnato libre 100. Que toccharà per uno? Fa così: Raccogli inseme tucto quello che ànno messo in compagnia, che sonno libre 50, et questo è'l corpo dela compagnia. Ora multiplicha per lo primo, che mise libre 10, et di': 10 via 100 libre fa 1000. Parti in 50, che ne vene 20 libre; et tanto toccha al primo. Ora multiplicha per lo secondo: 15 via 100 fa 1500 libre; parti in 50, che ne vene libre 30; et tanto toccha al secondo. Ora multiplicha per lo terzo: 15 via 100 libre ⟨~~fa 1500~~⟩, che fa ancho 1500 libre; parti in 50, anco' ne vene 30 libre; et tanto toccha al terzo. Ora multiplicha per lo quarto: 10 via 100 libre fa 1000 libre; parti in 50, che ne vene 20 libre; et tanto toccha al quarto. Et è facta. Et così se fanno le simiglianti ragioni.

[A1] A man rents a shop, and comes to stay there the first of January. Now comes another one and enters in company with him the first of April. Another one comes, and enters in company with them the first of July. Another one comes, and enters in company with them the first of October. The first puts in company, that is puts in the shop the first day he rented it, 100 *libre*. The second, on the day he entered company with them, put 200 *libre*. The third put 300 *libre*. The fourth put 400 *libre*. And thus all four stay together until the first of January. And at the end of the year they inspect their accounts. And the gain is found to be 100 *libre*. I ask you how ***(fol. 51^{r})*** this gain shall be divided, and what goes to one? You shall do thus: Each one puts on interest his money[31] for the time he has been in the company, at 2 *denari* the *libra* the month. And we say thus: The first has been in the company for a year, and put 100 *libre*, and shall have 10 *libre* in interest. And he who put 200, that it the second, has been in the company 9 months, and shall have 15 *libre* in interest. The third, who put 300 *libre*, has been in the company 6 months; he shall have 15 *libre* in interest. The fourth, who put 400 *libre* in company, shall have 10 *libre* in interest for three months. Now say thus: And there are 4 companions, who have made company together. And one puts in company 10 *libre*. And the other puts 15 *libre*. And the other puts 15 *libre*. And the other puts 10 *libre*. And they have gained 100 *libre*. What goes to one? Do thus: Collect together all that which they have put in company, which are 50 *libre*, and that is the principal of the company. Now multiply for the first, who put in 10, and say: 10 times 100 *libre* makes 1000. Divide by 50, from which results 20 *libre*; and as much goes to the first. Now multiply for the second: 15 times 100 makes 1500; divide by 50, from which results 30 *libre*; and as much goes to the second. Now multiply for the third: 15 times 100 *libre*,

[31] Since the currency which actually occurs in the problem is the *libra*, *denari* is obviously used here in the generic meaning of "money". Similarly in the following problem.

which again makes 1500; divide by 50, from which again results 30 *libre*; and as much goes to the third. Now multiply for the fourth: 10 times 100 *libre* makes 1000 *libre*; divide by 50, from which results 20 *libre*; and as much goes to the fourth. And it is done. And thus the similar computations are made.

Mathematical commentary: We observe how the standard company problem is used as a *functionally abstract* representation for a different problem of proportional division – thereby confirming the suspicion of such a use which one gets from the final part of [3] of the algebra section (cf. p. 9).

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After another group of mostly geometric problems, this question is found:

(Fol 58^v)
[...]
[A2] Egli è uno che fa compagnia con un altro. Et costui mette in su la bottegha una quantità di denari. Et quando vengono in capo dell'anno costui se trova avere guadagnato el terzo de quello che mise de capitale. Et ancora lo mise in compagnia sopra al capitale primo. Et poi in capo del secondo anno se trova avere guadagnato el quarto de ogni cosa, cioè che à in su la bottegha. Et anchora questo mette in su la bottegha sopra alli altri. Et poi in capo del terzo anno se trova avere guadagnato el quinto de ciò che à in su la bottegha. Et tra quello che vi misse de primo capitale et el guadagnio facto se trova avere in tucto in su la bottega fiorini 1200. Vo' sapere quanti denari misse de prima in su la bottegha. Fa chosì chomo in molte altre ragioni adietro abbiamo facto. Questa conviene se faccia propositione. Cioè che trove uno numero nel quale sia $^1/_3$ e $^1/_4$ e $^1/_5$, et questo numero è 60. Ora tu di' che'l primo anno guadagnò el $^1/_3$ de quello che vi mise. Sì che noi dirremo vi mettesse fiorini 60, guadagna el $^1/_3$, che è 20. Pollo sopra a esso, fa 80. Sì che el secondo anno mette in compagnia 80. Et tu di' che guadagnia el $^1/_4$, che vene a guadagnare 20; pollo sopra a esso, fa 100. Sì che'l terzo anno mette in compagnia 100. Et tu di' che guadagnia el quinto, che vene a guadagnare ancho 20; pollo sopra a esso, fa 120. Sì che a questo modo se trovarebbe tra'l capitale e'l guadagno in capo de tre anni 120 fiorini. Et noi diciamo che se trovò avere guadagniato fiorini 1200. Et però diremo così: per 60 fiorini che io me appongo me viene fiorini 120. Et io voglio me vengha fiorini 1200. Et però moltiplicha 60 via 1200, che fa 72000 de fiorini, parti in 120, che ne vene fiorino 600. Et cotanto ***(fol 59^r)*** mise di primo capitale in su la bottegha. Provala: El primo anno guagagniò el $^1/_3$, sonno 200 fiorini; ài che ebe fiorini 800. El secondo

guadagniò el $^1/_4$. Sonno ancho 200 fiorini, ài che ebe fiorini 1000. El terzo anno guadagnò el $^1/_5$, sonno ancho fiorini 200; ài che ebe in tucto fiorini 1200. Et sta bene et è facta. Et così se fanno le simili ragioni.

[A2] There is one who makes company with another. And he puts into the shop an amount of money. And when they come to the end of the year, this one finds to have gained the third of what he put in as capital. And again he put it in company in addition to the first capital. And then in the end of the second year he finds to have gained the fourth of everything, that is, of that which he put into the shop. And this again he puts into the shop in addition to the other (contributions). And then in the end of the third year he finds to have gained the fifth of that which he put into the shop. And with that which he put into it as the first capital and the gains he made he finds to have in all in the shop 1200 florins. I want to know how much money he put into the shop at first. Do thus as we have done in many other computations before. Here one ought to make a position. That is, that you find a number in which there is $\frac{1}{3}$ and $\frac{1}{4}$ and a $\frac{1}{5}$, and this number is 60. Now you say that in the first year he gained the $\frac{1}{3}$ of that which he put. So that we shall say that he put 60 florins, and gains the $\frac{1}{3}$, which is 20. Put it above that (amount), it makes 80. So that the second year he puts into company 80. And you say that he gains the $\frac{1}{4}$, then he turns out to gain 20; put it above that (amount), it makes 100. So that the third year he puts in company 100. And you say that he gains the fifth, then he turns out to gain again 20; put it above that (amount), it makes 120. So that in this way capital and gains together in the end of three years would be found to be 120 florins. And we said that he found to have gained 1200 florins. And therefore we shall say thus: for 60 florins which I posit 120 florins result for me. And I want that 1200 florins should fall to me. And therefore multiply 60 by 120, which makes 72000 florins, divide by 120, from which 600 florins result. And as much ***(fol 59^r)*** did he put as first capital into the shop. Verify it: The first year he gained the $\frac{1}{3}$, which are 200 florins; you get that he had 800 florins. The second year he gained the $\frac{1}{4}$. These are again 200 florins, you get that he had 1000 florins. The third year he gained the $\frac{1}{5}$, these are again 200 florins; you get that he had in all 1200 florins. And it goes well, and it is done. And thus the similar computations are made.

Mathematical commentary: We notice that the request for the position p that it contain a convenient $^1/_4$ and a convenient $^1/_5$ is only relevant because 3, 4 and 5 are mutually prime, since $^1/_4$ is to be taken of $p\cdot(1+^1/_3)$ and $^1/_5$ of $p\cdot(1+^1/_3)\cdot(1+^1/_4)$. In the present instance, the false position may be used quite mechanically.

Observations and inferences

As fourteenth-century vernacular algebras in general,[32] this one is interested in degrees higher than the second. In contrast to most other *abbaco* masters, however, Jacopo only lists cases he is able to solve, telling explicitly that they can be reduced to the second-degree cases (which of course is not literally true of the case $\alpha K = \beta$), and gives all solutions correctly.

When describing Paolo Gherardi's *Libro di ragioni*, Warren Van Egmond [1978: 163] commented thus upon the higher-degree problems of this work from 1328, equally written by a Florentine master, and this one certainly in Montpellier:

> The fact that this manuscript was written in Montpellier immediately suggests a possible connection with Arab Spain, but there are no extant Arab or Spanish documents with similar contents, nor are there any words or phrases which would suggest a foreign origin. In fact, the very crudeness of the document and the lack of mathematical understanding it displays leads me to question whether there could have been any Arab source. Surely no Arab algebraist could have written a treatise so full of elementary errors, and Leonardo Pisano was far too intelligent to have written such foolishness. Thus the very shortcomings of the text itself leads me to believe that it was composed by a relatively untrained and certainly naive European mathematician who perhaps thought he could extend upon al-Khwarizmi but was too ignorant to recognize his error. Whether it was Gerardi himself who did this or whether he took it over from some earlier writer is something we may never know.

By accident, we are now able to give a partial answer. Phrase-for-phrase comparison of some of Gherardi's problems with Jacopo's (most characteristically [9] and [13]) leave no doubt that either he or some source of his had Jacopo's treatise or something very close to it on his desk and used it, but did so "creatively". We may look at the beginning of the analogue of [9]:[33]

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[32] See, for instance, [Franci & Toti Rigatelli 1988].

[33] Ed. [van Egmond 1978: 167]; similarly [Arrighi 1987: 100]. In translation, using the same English equivalences as in the above translation of Jacopo's treatise):

> Somebody lends to another 20 *libre* for 2 years, to do at the end of year. When 2 years have passed he gives back 30 *libre*. I ask you, at which rate the *libra* is lent the month? Let us posit that the *libra* was lent for one thing the month. The *libra* is worth 12 things the year. For the 12 things take the $^1/_{20}$ and say

Uno presta a un'altro 20 libre in 2 anni a fare capo d'ampno. Quando viene in qua 2 anni eli rende 30 libre. Domandoti a che ragione e prestata la livre lo mese? Pognamo che fusse prestata la libre lo mese a una cosa. Vale la libre l'anno 12 cose. Per lo 12 cose pigla lo $^1/_{20}$ e di cosi: lo $^1/_{20}$ di 20 libre sie anche una cosa, ad ai per lo primo anno libre 20 e una cosa. Or fa lo secondo anno e di, lo $^1/_{20}$ di 20 libre sie anche una cosa e lo $^1/_{20}$ d'una cosa del primo anno sie $^1/_{20}$ di ciensο; ed ai ora 20 libre e 2 cose e uno $^1/_{20}$ di cienso. E abbiamo che 20 libre e 2 cose e $^1/_{20}$ di censo sono iguali a 30. Trahe 20 libre di 30, rimane 10. Dunque aviamo che 2 cose e $^1/_{20}$ di cienso sono iguali a 10 libre. Dovemo partire ne ciensi. Dovemo recare a uno cienso incasano per 20 e di, 20 via $^1/_{20}$ di cienso fa 1 censo, e 20 via 2 censi [*sic*, for cose] fa 40 cose, e 20 via 10 fa 200. Ora abbiamo che 4⟨0⟩ cose e uno cienso e iguali a 200. Dovemo dimezare le cose. Diremo la meta di 40 sie 20. Diremo 20 via 20 fa 400. Pone sopra 200 fa 600. Dunque diremo ch⟨e⟩ la radice di 600 meno 20, che fu lo dimezamento dele cose, valse la cosa. E tu ponesti che fusse prestata la libre lo mese a una cosa. Dirai una cosa via una cosa fa 1 in radice di 600 meno 20. Dunque a cotanto fu prestata la livra lo mese, cioe a ragione di ⟨radice di⟩ 600 meno 20 denari.

Gherardi, as we see, agrees with Jacopo's text step for step, although the formulations are often different. He even repeats the translation of "12 things" into "$^1/_{20}$" instead of "$^1/_{20}$ of thing"; but he omits the explanation of the unit in which the interest is measured – probably because interest was routinely expressed thus, but still a possible reason that he cannot explain well how 12 things become $^1/_{20}$; likewise omitted is the explicit observation that √600 is surd. The meaningless multiplication of 1 thing with 1 thing in the last step, exemplifies that "lack of mathematical

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thus: the $^1/_{20}$ of 20 *libre* is also a thing, and you have for the first year 20 *libre* and a thing. Now do the second year and say: the $^1/_{20}$ of 20 *libre* is also a thing, and the $^1/_{20}$ of a thing from the first year is $^1/_{20}$ of *censo*. And we have that 20 *libre* and 2 things and $^1/_{20}$ of *censo* are equal to 30. Detract 20 *libre* from 30, 10 remain. Then we have that 2 things and $^1/_{20}$ of *censo* are equal to 10 *libre*. We shall divide by the *censi*. We shall bring it to one *censo* by multiplying by 20, and say, 20 times $^1/_{20}$ of *censo* makes 1 *censo*, and 20 times 2 ⟨things⟩ make 40 things, and 20 times 10 makes 200. Now we have that 4⟨0⟩ things and one *censo* is equal to 200. We shall halve the things. We shall say that the half of 40 is 20. We shall say, 20 times 20 makes 400. Put above 200, it makes 600. Then we shall say that the root of 600 less 20, which was the halving of the things, is the thing. And you posited that the *libra* was lent the month for one thing. You shall say 1 thing times 1 thing makes 1 in root of 600 less 20. Then for as much was lent the *libra* the month, that is at the rate of ⟨root of⟩ 600 less 20 *denari*.

understanding" which Van Egmond speaks about – even if we assume that a writing error has crept in and read "1 {thing} times 1 thing", the step is superfluous.

A third text where the problem turns up is an *abbaco* manuscript from Lucca from c. 1330 [ed. Arrighi 1973: 195*f*]. The numbers are as Gherardi's, and so is the structure; the unit *denario* is omitted, and the being surd of √600 is not discussed. But there is no superfluous multiplication of thing by thing or by *1* in the end, and the critical passage in the beginning runs as follows:

> Ponj che fusse prestata a una chosa, l'anno vale 12 chose, per le 12 chose piglia il $^{1}/_{20}$ dj Libre 20 ch'è una chosa: ài libre 20 e 1 chosa. Ài a meritare Libre 20 e 1 chosa per un altro anno, che vale Libre 12 chose; piglia il $^{1}/_{20}$ di Libre 20 e 1 chosa ch'è 1 chosa e $^{1}/_{20}$ di cienso.[34]

From this problem alone we cannot exclude that the anonymous Lucca author depended on Gherardi and repaired the weaknesses of his original; but the treatment of the voyage problem (Jacopo's [13]) shows that both depend on a common ancestor who himself depended on Jacopo or a close relative, and which Gherardi maltreated:[35] both Gherardi and the Lucca treatise tell in the rule that there are two possible solutions, as does Jacopo; but Gherardi finds only one. In the Lucca treatise, moreover, the final sum of capital and gain is 64, which still yields nice integer solutions; Gherardi, taking the round sum 100, finds the unhandy answer 38+√1300. Similarly, the list of higher-degree problems in the Lucca treatise is an excerpt from Jacopo's list, and like Jacopo's not accompanied by examples; Gherardi's list of sometimes irreducible cases accompanied by wrong rules is provided with examples – but all of them are of the same kind (pure-number-problems, easily constructed so as to fit the case) – which, in its contrast to the variegated treatment of the second degree, suggests that all examples are due to the same hand.[36]

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34 "Posit that it was lent at one thing, in the year it is worth 12 things. For the 12 things take the $^{1}/_{20}$ of *libra* which is one thing: you have 20 *libre* and 1 thing. You have to put on interest 20 *libre* and 1 thing for another year, the *libra* being worth 12 things; take the $^{1}/_{20}$ of 20 *libre* and 1 thing, which is 1 thing and $^{1}/_{20}$ of *censo*".

35 Actually, as we shall see (p. 51), Gerardi himself is not likely to have been the one who introduced the fallacious innovations; between the archetype of the Lucca treatise and Gerardi, another lost work will have to be inserted, where the deceptive rules and their examples were introduced.

36 What was said here about the Lucca algorism refers to the section "Algebra", fols 80^{v}–81^{v}. Fols 50^{r}–52^{r} contain an analogous section "Regola della cosa". (The

It is thus evident that Gherardi is neither the first nor the second but at least the third link (rather the fourth) in a chain where Jacopo or some close model of his is the first. The weaknesses, on the other hand, turn out to be secondary developments. It should therefore be legitimate to return to the question whether the interest of the fourteenth-century abbacists in higher-degree problems (and possibly their algebra in general) might be derived, either from direct inspiration from the Arabic world or, rather, indirect inspiration perhaps by way of Catalan-Occitan commercial calculation[37]), and only marginally from autochthonous transformations of a tradition going back to the *Liber abaci* – not least because all Jacopo's higher-degree cases were routinely solved in Arabic algebra. On the other hand, Van Egmond's suspicion that the fallacies are "Christian" inventions is confirmed beyond doubt.

That marginal influence either from the *Liber abaci* or from Gerard of Cremona's Latin translations is present seems plausible on terminological grounds: *Census*/*censo* is hardly the most obvious translation of Arabic *māl*.[38] For the remaining components of *abbaco* mathematics (the algorism and the non-algebraic first-degree problems), Leonardo's importance is certainly not to be doubted; indeed, the earliest *Livero de l'abbecho* known so far, an Umbrian specimen from the second half of the thirteenth century, presents itself as derived from "la oppenione de maiestro Leonardo de la

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treatise is a conglomerate written by several hands). The two treatments of the topic are independent of each other, and parallel descendants from the same source. Apart from details, all that was said about the "algebra" section holds for the "cosa" section.

[37] The total absence of Arabisms from Jacopo's work (unless we count the *fondicho* or *fundecho*, derived from Arabic *funduq*) speaks in favour of an indirect connection through a Romance-speaking environment. Since Catalan trade was mainly oriented towards the Arabic world in the later thirteenth century [Abulafia 1985], a Catalan-Occitan connection will also be more plausible than a direct channel to the Arabs on the part of a Florentine *abbaco* master working in Montpellier.

It may be significant that a Provençal commercial arithmetic from c. 1430 described by Jacques Sesiano [1984] also refers to its subject as *algorismo*, not as *abbaco* as most Italian treatises would do. Jacopo's source for the word need not be the Latin algorisms.

[38] However, it is also used in the *Liber augmenti et diminutionis* [ed. Libri 1838: I, 304–371], in which it refers to an unknown amount of money (Libri's and later commentators' translation as x^2 notwithstanding). This work is not algebraic but solves all its problems (invariably of the first degree) by single or double false position.

chasa degli figluogle Bonaçie da Pisa" [ed. Arrighi 1989], and does so in full right. But this work contains no algebra.

The rules for the higher degrees are not the only features that suggest Jacopo to owe little to Leonardo, and which speak in favour of links to the Arabic world. First of all one notices the absence of geometric proofs, present not only in the *Liber abaci* and in Fibonacci's *Pratica geometrie* but also in virtually all Latin algebras: Gherardo of Cremona's, Robert of Chester's and the anonymous translation of al-Khwārizmī, and the anonymous translation of Abū Kāmil.[39] The sole exceptions are the embedded *al-jabr* solutions in Abū Bakr's *Liber mensurationum*, which only makes use of the rules, referring to a preceding introduction of the topic where they will have been enunciated explicitly; and the brief and ineffectual presentation of "gleba mutabilia" in *Liber Alchorizmi de pratica arismetice*.[40] As pointed out by Raffaella Franci and Laura Toti Rigatelli [1988: 18], this absence of geometric proofs is a general characteristic of fourteenth-century vernacular algebra, to which they have located only two exceptions from the very end of the century.

A second noteworthy feature is Jacopo's consistent reference to non-normalized cases – all his rules, we notice, start by normalizing the equation. In this respect, Jacopo is followed by the bulk of *abbaco* algebraists of the century. Even this is in contrast to the enunciation of the second-degree rules in all Latin treatises (this times without exceptions),[41] and certainly to the originals.[42] Only al-Karajī, in the *Kāfī*, represents all three

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39 Ed., respectively, [Hughes 1986], [Hughes 1989], [Kaunzner 1986], and [Sesiano 1993].

40 Ed., respectively, [Busard 1968] and [Boncompagni 1857b: 112*f*]. The algebra section is not in Allard's partial edition of the *Liber Alchorizmi* [1992] but present in manuscripts that are as far as possible from each other in the stemma – see [Høyrup 1998b: 16 n.7] and thus doubtless part of the original work and no interpolation.

41 The texts certainly teach how to normalize non-normalized problems, but this is not part of the standardized *rule*. The first-degree problem is evidently never normalized – otherwise the enunciation would *be* the solution.

42 The headings of the published Arabic manuscript of al-Khwārizmī's *Algebra* [ed. Mušarrafa & Aḥmad 1939] refer to the non-normalized cases, but given the grammatical faithfulness of Gerard of Cremona we may be certain that this is an innovation; as argued in [Høyrup 1998a], the extant Arabic manuscript is the outcome of at least three consecutive revisions.

simple cases in non-normalized form.[43]

A third feature of greater significance than has henceforth been noticed is the order in which the cases are listed (to my knowledge, Ahmed Djebbar is the only scholar who has observed that this might be an interesting parameter). In the translations of al-Khwārizmī and Abū Kāmil, the order is (*C* = *census*, *r* = *radix*, *n* = *numerus*): (1) $C = \beta r$, (2) $C = n$, (3) $\alpha r = n$, (4) $C+\alpha r = n$, (5) $C+n = \beta r$, (6) $\beta r+n = C$. The corresponding order in the *Liber abaci* is 1–2–3–4–6–5. Jacopo's order is 3–2–1–4–5–6. This order is followed by Gherardi and the Lucca manuscript (both times).

The Latin al-Khwārizmī / Abū Kāmil order corresponds not only to the Arabic originals[44] but also to what is found in most of the classical Arabic treatises – we may point to Thābit ibn Qurrah's *Verification of the Problems of Algebra through Geometrical Demonstrations* (ed., trans. [Luckey 1941: 105–107] – only 4–5–6); ibn al-Bannāᵓ's *Talkhīs*;[45] ibn al-Yāsamīn's ᶜ*Urjuza fi'l-jabr wa'l-muqābalah* (paraphrase in symbols in [Souissy 1983: 220–223]); and ibn Turk [ed. Sayılı 1962: 145–152] (1–4–5–6 only).

However, al-Karajī – both in the *Kāfī* [trans. Hochheim 1878: III, 10–13] and in the *Fakhrī* [paraphrase Woepcke 1853: 64–71] – has the sequence 3–1–2–4–5–6; he is followed by al-Samawᵓal, by al-Kāšī and by al-ᶜĀmilī – and even by ibn al-Bannāᵓ in his actual order of solving the equations [Djebbar 1981: 60*f*]. In al-Māridīnī's commentary to ibn al-Yāsamīn's ᶜ*Urjūza* from c. 1500 [Souissy 1983: 220], moreover, Jacopo's arrangement is mentioned and told to be what is used in "the East", and it is actually the order of al-Miṣṣīṣī, al-Bīrūnī, al-Khayyāmī and Šaraf al-Dīn al-Ṭūsī [Djebbar 1981: 60]. Since neither Jacopo nor anybody from his environment are likely to have come in touch with these authors, they must be presumed to agree with a more widespread practice of their times; and indeed, theirs and Jacopo's ordering is also found with al-Qurašī (thirteenth-century, born in Andalusia, active in Bugia in Algeria) [Djebbar 1988: 107].[46]

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43 Trans. [Hochheim 1878: III]. The composite cases are represented by an example only and without a general formulation. According to Woepcke's paraphrase [1853], the cases in the *Fakhrī* seem to be presented in non-normalized form, but since I only know this paraphrase I am not certain whether this be due to Woepcke or to al-Karajī.

44 Ed. [Mušarrafa & Aḥmad 1939: 17–21] and [Hogendijk 1986: 5], respectively

45 Ed., trans. [Souissi 1969: 92]; the cases are not listed, but internal references shows the presupposed order to be 1–2–3–4–5–6.

46 Djebbar links the changing order to the emergence of general polynomial algebra, thereby provoking the question whether the schematic polynomial algebra that

To this we may add, firstly, that not a single one of Jacopo's problems in the algebra section is shared with the *Liber abaci*. In the sole problem where the structure recurs (*viz* [12]), the set of numerical parameters is different. And, secondly and finally, that Jacopo's use of "restoration" (covering both the "restoration" and the "opposition" of the tradition going back to al-Khwārizmī and Abū Kāmil) is not far from al-Karajī's usage in the *Fakhrī* nor from what we find in Abū Bakr's *Liber mensurationum*.[47]

Al-Karaji was a brilliant mathematician, but much in his work shows that his initial point of reference was the "low" level of practical mathematics, not the al-Khwārizmī-Abū-Kāmil tradition.[48] In his advanced work he would certainly appropriate all he wanted from the "high" tradition, but his *Kāfī* remains one of the best witnesses we possess of the "low" current of Arabic algebra, of which all too little is known. To what was said above we may add that it contains no geometric proofs of the composite second-degree cases.

Given the sometimes partial, sometimes complete agreements with al-Karajī and the complete lack of positive correlation with Fibonacci, we are led to the following sketchy conclusions:

(i) Fourteenth-century *abbaco* algebra owes little to the *Liber abaci*,

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turns up in Canacci's *Ragionamenti d'algebra* [ed. Procissi 1954: 317–323] and Stifel's *Arithmetica integra* [1544: 238ʳ–239ʳ] (and which was borrowed immediately by Scheubel, Ramus and Lazarus Schoner) is somehow connected to the polynomial algebra of the Arabic world. On the surface, Jacopo's algebra seems to be purely rhetorical – but the way in which he speaks about the coefficient of the roots (etc.) simply as "the roots" both before and after normalization could be taken to suggest that his rules refer to a *position in a scheme* and not to any particular member of the sequence of numbers which come to occupy this position successively.

47 See also [Saliba 1972] on what seems to be the original (and thus pre-al-Khwārizmīan) use of the terms *al-jabr* and *al-muqābalah*.

Jacopo's usage recurs in the "algebra" but not in the "cosa" section of the Lucca manuscript; nor is it found with Gerardi.

48 Cf. also [Høyrup 1997, *passim*] on his practical geometry, which turns out to be much closer to the practical tradition than, e.g., Abū Kāmil's treatise on the same topic [ed. Sesiano 1996].

The distinction between a "low" and a "high" level is not meant to imply that information etc. did not move back and forth between the two. For any attempt to trace the development of algebra it is important to notice, however, that Arabic algebra is much more than what we find with the famous authors, and that much of what was in current use even in the early second millennium depended more on the pre-al-Khwārizmīan art than on al-Khwārizmī and Abū Kāmil.

whose algebra was only to gain some influence in the fifteenth century,[49] nor to the Latin translations of al-Khwārizmī and Abū Kamil.

(ii) Instead, the new start was inspired by direct or indirect contacts to the "low" level of Arabic algebra, still faithfully reflected by Jacopo, who *may* have been a main responsible for the borrowing into Italian (thought hardly directly from the Arabic) and is in any case close to it; the return to a more orthodox use of the term "restoration" already in sources from the subsequent decades shows that he cannot be alone.[50]

(iii) This beginning sparked off a fresh development in the abbacus school environment, expressed most clearly in the proliferation of rules for the higher degree – and characterized by ever-diminishing care for solutions being rational. It is not to be excluded that the two developments belong together: if the solution was an unhandy surd the temptation to control its validity would be modest.

A complete inventory of the problem types present in Jacopo's and other fourteenth-century writings on algebra and mapping of their antecedents will certainly allow us to produce a more detailed picture. This, however, falls outside the scope of the present article.

Closest kin

Instead, we shall have a brief look at the *Trattato dell'alcibra amuchabile*, an anonymous treatise known from a sole manuscript (Ricc. 2263, fols 24^r–50^v) from c. 1365. The following is based on Annalisa Simi's edition [1994].

The treatise consists of four parts. The first contains the rules for multiplying signed entities and binomia, starting thus:

> In prima dicho che più via più fa più e meno vie meno fa più et più vie meno fa meno et meno vie più fa meno et cosa vie cosa fa cienso e cosa vie cienso fa chubo e cienso vie cienso fa cienso di cienso e cienso vie chubo fa cienso

[49] Paradoxically, the only mid-fourteenth-century author who betrays familiarity with Fibonacci's algebra is thus a French scholarly mathematician and astrologer – *viz* Jean de Murs, see [L'Huillier 1980, *passim*].

[50] Not least because this orthodoxy is made superfluous by a development that is attested in those same sources: the emergence of a general notion of polynomium members that may be additive or subtractive.

di chubo e cosa vie chubo fa cienso di cienso.

Ora ti voglio insegniare multiprichare numero vie radicie d'un altro numero [...].[51]

The multiplication of binomials is taught in schemes. Although everything which is explained (apart from the outcome of the multiplication of *censo* with *chubo*) is used by Jacopo, there is nothing to suggest a connection.

This is followed by three pages (26ᵛ–27ᵛ) dedicated to non-mathematical subjects – another indication that the first part is wholly unconnected to the second, which turns out to coincide exactly with Jacopo's algebra, [1]–[14]. Spellings are different, it is true, but apart from that the texts are so close to each other that lacunae in one can be filled out by means of the other. At closer inspection it turns out, however, that the relationship between the two is not symmetrical. In Jacopo's [9], a passage has been overlooked by the copyist, cf. note 21. On the other hand, if we compare the final passage of his [13],

> Et abi a mente questa regola. In bona verità vorrebbe una grande despositione; ma non mi distendo troppo però che me pare stendere et sc'vere [scrivere?] in vile cosa; ma questo baste qui et in più dire sopra ciò non mi vo' stendere

with the corresponding lines in the anonymous treatise [ed. Simi 1994: 27],

> Ed abi a mente che questa reghola vorebe una grande dispusitione, ma non mi ci distendo tropo che melo pare scrivere multa cosa e questo basti[52]

it becomes clear from the compression of the first two sentences into one that the anonymous writer (or somebody from whom he copies) has used Jacopo (or something very close to his text) and tried to improve it. The

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51 "First I say that plus times plus makes plus and less times less makes plus and plus times less makes less and less times plus makes less and thing times thing makes *censo* and thing times *censo* makes *cubo* and *censo* times *censo* makes *censo* of *censo* and *censo* times *cubo* makes *censo* of *cubo* and thing times *cubo* makes *censo* of *censo*.

Now I will teach you to multiply a number times the root of another number [...]".

52 In translation, Jacopo's version is

> And keep in mind this rule. Verily, a vast exposition would be needed; but I will not enlarge too much, because I seem to expand and write about base things; but this should be enough here, and I will not enlarge more upon it

whereas the anonymous *Trattato* has

> And keep in mind that this rule would need a vast exposition, but I will not enlarge too much, as it seems to me I have already written many things, and this should be enough.

end suggests that whereas Jacopo is tempted to elaborate the argument, the anonynous writer finds his source too loquacious.

The third part contains the rules for the third and fourth degree. Nos 7–14 (including several irreducible cases solved by incorrect rules) are provided with illustrating examples; with one exception and a slight deviation, both the rules and the examples agree with Gherardi's *Libro de ragioni*. The deviation is a case where a numerical parameter is different; the exception concerns the reducible case "*cubi* and things equal *censi*", which is not in Gherardi's treatise (instead, it has "*cubi* are equal to things and *censi* and number"), but which is provided with an illustrative example of the same kind as Gherardi's; this is the reason that Gherardi himself is not likely to be the one who introduced the innovations, cf. note 35.[53]

Nos 15 to 24 are rules for reducible third- and fourth-degree cases deprived of accompanying examples; all are found with Jacopo in identical form (and the absences pointed out in note 26 are repeated), but several are absent from both algebra sections of the Lucca treatise.

The fourth and final part is a collection of miscellaneous problems, some which exhibit affinities to the Lucca treatise, others to Jacopo (at points where these two do not coincide), still others to none of them. They are likely to refer to a common stock of problems and illustrate that direct parentage between the various *abbaco* treatises (not to speak about some *abbaco* treatise and a work from the Arabic world or some more remote region) cannot be argued convincingly just from one or a few shared problem types. Any investigation of such questions (and thus, *a fortiori*, of originality versus borrowing) should pay close attention to numerical parameters; to the steps of the computation and their order; to the structure of phrases and to the choice of words and grammatical forms; to the ordering of the sequence of problems; and to omissions and innovations on all levels.

Until this day, that kind of work has hardly begun; the preceding pages may serve to illustrate its possibilities (and the pitfalls), but they go no further. Thanks to the array of manuscripts which have been published by Gino Arrighi and two generations of scholars inspired by him, fairly adequate material is now at hand as far as the Italian vernacular scene is concerned; for understandable reasons (but also because the sources that are needed have low prestige) it is still sorely lacking on the Arabic side,

[53] In some of the cases which agree with Gerardi, the precise wording and grammar of the rule (which, as we remember, is given without illustration by Jacopo) is often closer to Jacopo than to Gerardi.

although the recent generation of Maghreb historians of mathematics have begun the work. As far as the Catalan-Occitan scene before Chuquet is concerned, Jacques Sesiano [1984] has made it clear that it deserves to be explored; but little has been done beyond that.

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