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A NOTE ON OLD BABYLONIAN COMPUTATIONAL TECHNIQUES

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ABSTRACT

Analysis of the errors in two Old Babylonian "algebraic" problems show, firstly, that the computations were performed on a device where at least additive contributions were no longer identifiable once they had entered the computation; secondly, that this device must have been some kind of counting board or abacus where numbers were represented as collections of calculi; and, thirdly, that units and tens were represented in distinct ways, perhaps by means of different calculi.

Eine Analyse der Rechenfehler in zwei altbabylonische "algebraische" Aufgaben zeigen, erstens, daß die Berechnungen auf einem Gerät durchführt wurden, wo wenigstens additive Beiträge nicht länger identifizierbar waren nach ihrer Eintragung in die Rechnung; zweitens, daß das Gerät irgendeine Art Rechenbrett war, wo Zahlen als Haufen von Rechensteinen erschienen; drittens, daß Einer und Zehner in verschiedener Weise, vielleicht mittels verschiedener Rechensteine repräsentiert wurden.

It has been known for more than a century that Babylonian calculators made use of tables of multiplication, reciprocals, squares and cubes. It is also an old insight that such tables alone could not do the job – for instance, a multiplication like that of 2 24 and 2 36[1] (performed in the text VAT 7532, obv. 15, ed. [Neugebauer 1935: I, 294]) would by necessity require the addition of more partial products than could be kept track of mentally, even if simplified by means of clever factorizations. It has therefore been a recurrent guess that the Babylonians might have used for this purpose some kind of abacus – Kurt Vogel [1959: 24] also pointed to the possibility that the creation of the sexagesimal place value system might have been inspired by the use of a counting board.

Denise Schmandt-Besserat's discovery [1977] of the continuity between an age-old accounting system based on clay tokens and the earliest cuneiform writing could only give new life to such speculations, the fullest development of the argument being probably [Waschkies 1989: 84*ff*]. Unfortunately, neither material finds nor texts allowed to transform the speculations into something more substantial – as pointed out explicitly by Waschkies [1989: 85], no document was known at the time which contained intermediate calculations or which told in clear terms how they were made.

Old Babylonian documents containing "rough work" were only identified by Eleanor Robson ([1995], republished in final form as [Robson 1999]; further examples, e.g., in [Robson 2000]).[2] What we learn from these is, however, that calculations whose result could not be found by mental calculation (after adequate training) were performed in a different medium; thus, the tablet UET VI/2 222 states directly (and correctly) that 1 03 45 times 1 03 45 (expressed by the writing of one number above the other) is 1 07 44 03 45. Since none of the round tablets discussed by Robson contains the details of such calculations, we must presume that they were not made in clay. How they were then made remains an open question.

Fortunately, not all calculators are equally precise, and calculational errors in the sources may often be as informative about the process in which they were produced as those made in the class-room may be about the way school kids think about mathematical objects. Errors contained in two (equally Old Babylonian) texts belonging to the so-called "algebraic" genre turn out to shed some light on the nature of the devices of which their authors made use.

The first is problem no. 12 of the tablet BM 13901 (obv. II, lines 27–34, ed.

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[1] Since the present discussion regards calculation within the sexagesimal floating-point place value system, I render the numbers without any indication of a presumed absolute order of magnitude. 2 24 thus stands for $2 \cdot 60^{n}+24 \cdot 60^{n-1}$, where n can be any integer.

[2] The Old Babylonian period lasts from 2000 BCE to 1600 BCE in the currently used "middle chronology".

[Neugebauer 1935: III, 3]). Line 29 asks for the multiplication of 10´50˝ by 10´50˝,[3] and line 30 states the result as 1´57˝46‴40$^{(4)}$ – wrongly, indeed, the true answer being 1´57˝21‴40$^{(4)}$. Since the erroneous result is used further on, it must be due to the author of the text, not to a copyist. The computation can be made in many ways, but I have been able to figure out only one where 25 occurs as an intermediate result, and indeed in the order of thirds: a determination of the partial product 50˝· as 25·(10˝·10˝) = 25·(1‴40$^{(4)}$) = 25‴+16‴40$^{(4)}$.

1	57	***21***	40
1	57	***46***	40

This does not inform us about the tool on which the computation was performed, but is interesting in itself. The use of factorization agrees well with what is known from other sources – thus for instance from the tablets for rough work published by Eleanor Robson. But factorization only intervenes when the answer cannot be given immediately. We thus discover that the author of the tablet knew by heart (or because he had just made use of it for the determination of another partial product) that 10·10 = 1 40; but he seems not to have known by heart that 10·50 = 8 20, nor that 50·50 = 41 40. (Since the details of the computation are not presented in the text, a pedagogically motivated detour can be safely excluded).

	10	50 × 10	50
01	40		
	05		
	03	20	
	05		
	03	20	
		25	
		25	
		16	40
01	57	46	40

The fact that the contribution 25‴ is added twice does tell us something about the calculational tool. Omission of a contribution can occur in almost any kind of device. Insertion twice instead of once, on the other hand, is next to excluded if the single contributions remain visible to the reckoner, as in our paper algorithms (anybody going from paper to the pocket calculator will have experienced the unpleasant change on this account). We must therefore conclude that our Old Babylonian calculator operated in a medium where at least additive contributions were no longer identifiable once they had entered the computation – as in the medieval dust abacus or on a counting board, but not in the paper algorithms that were developed by the late medieval *maestri d'abbaco* and which are still with us.

Further information is obtained from the second problem of the text TMS XIX

[3] Actually the text asks for the laying-out of a rectangle with these sides and the ensuing determination of the area. However, the present inquiry concerns only the numerical aspect of the question, for which reason it will be convenient to disregard the geometrical setting.

Since the analysis requires that the relative order of magnitude of members be kept clear, from this point onward I make use when adequate of Thureau-Dangin's extension of the degree-minute-second notation. It should be kept in mind that the tablet contains no similar indications.

(rev., lines 1–12, ed. [Bruins & Rutten 1961: 103, pl. 29].[4] In line 4, the square on 14´48˝53‴20$^{(4)}$ is determined as 3´39˝[28‴]44$^{(4)}$26$^{(5)}$40$^{(6)}$,[5] and not as 3´39˝28‴43$^{(4)}$27$^{(5)}$24$^{(6)}$26$^{(7)}$40$^{(8)}$). Two errors appear to have been committed: first, 43 27 has become 44 26; next, the repetition of «26» (in both cases preceded by «4») has made the calculator change «44 26 24 26 40» into «44 26 40». Since the number that is produced is used further on, even the latter error must have been committed by the original calculator when transferring it from a separate device on which it had been found. The first error – the misplacement of a single unit in a wrong order of magnitude – implies that numbers were represented as collections of units in the calculational device, in the manner of calculi placed on a counting board, and not as the written numbers on a dust abacus.

3	39	28	4***3***	2***7***	24	26	40
3	39	28	4***4***	2***6***	24	26	40
3	39	28	44			26	40

In lines 6–7, 11˝6‴40$^{(4)}$ is added to the number 3´39˝[28‴]44$^{(4)}$26$^{(5)}$40$^{(6)}$,[6] and the result is stated to be 3´50˝36‴43$^{(4)}$34$^{(5)}$26$^{(6)}$40$^{(7)}$ instead of 3´50˝35‴24$^{(4)}$26$^{(5)}$40$^{(6)}$. This error is more complex, and since the number is not used further on[7] we cannot know whether a copyist's error (or an unsuccessful copyist's attempt to repair a recognized error) has been superimposed upon an original calculator's error. It seems, however, that a unit has been misplaced in the order of fourths instead of that of thirds; besides, two tens have been added wrongly to the fourths, and an extra place «34» inserted after the

3	50	3***5***	2***4***		26	40
3	50	3***6***	2***3***		26	40
3	50	36	43		26	40
3	50	36	43	34	26	40

[4] In the problem, the area of a rectangle is given together with the area of another rectangle, whose length is the cube on the length of the first rectangle, and whose width is the diagonal of the first rectangle. This is a problem of the eighth degree, which is solved correctly (apart from the calculational errors which are discussed below) as a bi-biquadratic.

[5] Bruins's transliteration has «3.39.2[8.43.27]⟨24⟩.26.40», but Ruttens's hand copy shows that there is space for nothing more than 28, and that the presumedly missing «24» is present but as «44»; moreover, the number is repeated in line 7 as «3´39˝28‴44⁗26$^{(5)}$[40$^{(6)}$]».

There is no photo of the tablet in the edition, which means that this is one of the tablets that were mislaid by the Louvre after having been hidden away in the late thirties because of fear of imminent war [Jim Ritter, personal communication]; Bruins will therefore have had to make his transliteration from the hand copy, and cannot have improved the readings of the latter after collation.

[6] Bruins gives the number as «3.39.28.44.26.24.[26.40]»; however, according to the hand copy only «3.39.28.44.26» is legible, and the final lacuna has space for only one sexagesimal place – i.e., exactly for the number found in line 4. Bruins has evidently reconstructed with an eye to the correct value.

[7] Its square root is taken in lines 8–9, but the stated value «15´11˝6‴40⁗» is obviously found from the known end result, which is the reason that the solution can be stated correctly. The same, by the way, happens in BM 13901 no. 12.

fourths.

Since misplaced units appear not to turn up as tens, it is likely that counters for units and tens were different (as are the corresponding cuneiform signs, and as would also be expected if the pre-literate accounting system had provided the original inspiration); alternatively, a counting board may have been in use where cases (or carved grooves, or whatever was used to keep together counters that belonged together) for units and for tens were clearly distinguished but cases for units in neighbouring orders of magnitude were so spatially close that single counters could be mislaid or pushed accidentally from one to the other.

This is as far as the errors can bring us. Other textual evidence is at best ambiguous.[8] Archaeology only tells us is that the implements in question will either have been made of perishable materials or have been of a type that has not allowed archaeologists to identify their function.

It may be added that the use of a counting board will explain the rarity of mistaken place ascriptions in the mathematical texts, for instance in the addition of multi-place numbers. Mistakes are, indeed, much less common than could be expected if absolute orders of magnitude were to be kept track of mentally, without any material support.

It may also be added, and should be emphasized, that all conclusions drawn above were based upon Old Babylonian material. There is no certainty that similar techniques were used in the first millennium BCE. By then, the wax tablet had come into use. A dust abacus is also likely to have been employed, as revealed by the Greek name for the abacus (αβαξ): as first pointed out by Nesselmann [1842: 107 n.5], it is a West Semitic loan word, derived from a verb meaning "to fly away" and/or from a cognate noun meaning "light dust".

[8] Stephen Lieberman [1980: 346*f*], it is true, supposed to have found terms in lexical lists that designated wooden accounting and calculational devices. His key piece, however, was a first millennium version of one such list; the Old Babylonian version of the same list which has been found in the meantime does not corroborate his interpretations (Eleanor Robson, personal communication).

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