

Jens Høyrup

Lengths, Widths, Surfaces

A Portrait of Old Babylonian
Algebra and Its Kin

With 89 Illustrations



Springer

Jens Høyrup
Section for Philosophy and Science Studies
University of Roskilde
P.O. Box 260
DK-4000 Roskilde
Denmark
jensh@ruc.dk

Sources and Studies Editor:
Gerald J. Toomer
2800 South Ocean Boulevard, 21F
Boca Raton, FL 33432
USA

Library of Congress Cataloging-in-Publication Data
Høyrup, Jens.

Lengths, widths, surfaces : a portrait of old Babylonian algebra and its kin / Jens Høyrup.
p. cm. — (Studies and sources in the history of mathematics and physical sciences)

Includes bibliographical references and index.

ISBN 0-387-95303-5 (alk. paper)

1. Mathematics, Babylonian. 2. Algebra. I. Title. II. Sources and studies in the history of mathematics and physical sciences.

QA22 .H83 2001

510.935—dc21

2001032839

Printed on acid-free paper.

© 2002 Springer-Verlag New York, Inc.

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden. The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Timothy Taylor; manufacturing supervised by Erica Bresler.

Photocomposed copy prepared from the author's files.

Printed and bound by Maple-Vail Book Manufacturing Group, York, PA.

Printed in the United States of America.

9 8 7 6 5 4 3 2 1

ISBN 0-387-95303-5

SPIN 10838641

Springer-Verlag New York Berlin Heidelberg
A member of BertelsmannSpringer Science+Business Media GmbH

To all the Assyriologist-friends in Copenhagen,
Leningrad, Illinois, and Germany East and West
who never refused assistance;

to Peter, Jöran, and Jim;

and in memory of O. Neugebauer

*ogni approfondimento di ricerca rivela una
complessità di elementi dei quali precedenti
sintesi non avevano tenuto sufficiente conto, e,
se li avevano presi in considerazione, non
era sembrato che infirmassero una tesi di
primaria importanza, di solito condizionata
dal gusto imperante al tempo in cui venne
formulata quella sintesi.*

Mario Praz, *Gusto neoclassico*

Preface

“[...] it is through wonder that men now begin and originally began to philosophize” – thus Aristotle’s *Metaphysica* 982^b12 [trans. Tredennick 1933: I, 13]. Some 25 years ago *I* started wondering when reading the secondary literature about the early history of mathematics: what could be the reasons that induced the Babylonians to work on second-degree equations, as it was said they did? Obviously not practical applicability – nor, however, it appeared, that kind of curiosity which made the ancient Greeks create mathematical *theory*.

In parallel with many other questions, I pursued the matter until I believed – around 1980 – to have arrived at least at a rough explanation. Looking at what I wrote back then I can still recognize the inception of my present ideas about the historical sociology of Babylonian mathematical knowledge; but beyond the general Assyriological literature, my basis consisted of translated sources whose interpretation had been commonly accepted since the 1930s.

In 1982 I gave a guest lecture in Berlin on my sociological interpretation, after which a member of the audience asked me *what* this Babylonian algebra looked like. I answered in agreement with what I had understood on the basis of the translations, and thus gave a picture close to the rhetorical algebra of the Middle Ages. Peter Damerow, who had organized the session, at that moment asked me why I was so sure, and showed a geometrical interpretation which Evert Bruins had proposed for a particular text; I recognized the diagram from one of the geometrical proofs from al-Khwārizmī’s *Algebra* (it is shown below in Figure 88, p. 413), which made me curious. I got hold of a grammar and a dictionary and soon realized that the diagram was totally irrelevant in the context where Bruins had used it; but I also discovered that the current interpretation of the Babylonian “algebraic” texts was made to fit the numbers but did not agree with what followed from a careful reading of the words between the numbers.

For the outsider, Assyriology comes close to being an occult science, and it took some years before I was able to publish a decent detailed account of

my arguments and my results [Høyrup 1990]. That I got so far was largely due to the support I got from Bendt Alster, Mogens Trolle Larsen, and Aage Westenholz of the Carsten Niebuhr Institute, University of Copenhagen, and to the discussions I had with the participants in the “Workshops on Concept Development in Babylonian Mathematics” organized in Berlin in 1983, 1984, 1985, and 1988 – especially with Peter Damerow, Robert Englund, Jöran Friberg, Hans Nissen, Marvin Powell, Johannes Renger, and Jim Ritter. I also got precious advice and patient encouragement from Wolfram von Soden, even though it took me years to convince him that I might be on the right track.

That I got further is thanks to the colleagues who prevented me from concentrating all my scholarly energies on Mesopotamia, and seduced me into pursuing parallel work on ancient Greek, Islamic, and Latin medieval mathematics. Though I started in the likeness of Columbus, hitting land on a course I had initially chosen for the wrong reasons, I continued rather like Odysseus, visiting many unfamiliar countries, staying long with Circe and with Calypso. I also lost some experienced companions and masters on the way whom I think of with much regret – first Kilian Butz and Kurt Vogel, more recently Wolfram von Soden and Wilbur Knorr; I even visited the realm of the dead and learned immensely from the shadows of Thureau-Dangin and Neugebauer.

I was never left alone on the shore of Ithaca (if that is where I am now), but the possessions on my shelves are no less precious for me than the gifts of the Phaeacians for Odysseus: books, articles, letters from colleagues, and my own notes and writings on many intersecting themes. The pages that follow build on these riches, synthesized as far as I can at the present stage. The core of the argument is an analysis of the techniques and conceptualizations of Babylonian “algebraic” and related mathematics from the “Old Babylonian” earlier second millennium BCE (the “golden age” of Babylonian mathematics), based on texts in transliteration and “conformal translation”; on this foundation, a global portrait of the mathematical type in question is delineated.

These are the topics of Chapters I–VII. They deal with a moment in the history of mathematics, but the approach is not historical: it is synchronous and does not ask about the development nor, *a fortiori*, about the forces that shaped this development. The rest of the book (Chapters VIII–XI) is devoted to history proper: the historical shaping of Old Babylonian mathematics itself, the detailed geographical and chronological pattern; the origins and transformation; and, finally, kinship and historical influence.

I shall abstain from reformulating in prose what may just as well be read from the table of contents, and close this *prolegomenon* with three technical remarks:

All translations into English in the following – both from the sources and from modern publications – are mine, if no other translator is identified.

References mainly follow the author/editor-date system (with alphabetization after first author in the bibliography, pp. 418ff). However, standard editions of Babylonian texts and Assyriological reference works are

referred to by the customary abbreviations, which are also listed in the bibliography.

Babylonian tablets are referred to by habitual museum or publication numbers. The “Index of Tablets” (pp. 426ff) inventories all tablets referred to in the text and refers to the publications from which I have taken the single texts. It also lists the references to each text in the preceding pages.

Jöran Friberg read and commented valuably on part of the first draft and Eleanor Robson on the second version, for which I thank both sincerely. I hardly need to point out that I remain responsible for everything, both where I have followed their suggestions more or less faithfully and where I have decided differently.

Contents

PREFACE	vii	
CONTENTS	xi	
I	INTRODUCTION	1
	The Discovery of Babylonian “Algebra”	1
	The Standard Interpretation	3
	The Texts, the Genre, and the Problems	8
II	A NEW READING	11
	An Example	11
	Structural Analysis and Close Reading	14
	Numbers and Measures	15
	Mathematical Operations	18
	Additive Operations (19); Subtractive Operations (20); “Multiplications” (21); Rectangularization, Squaring, and “Square Root” (23); Division, Parts, and the <i>igi</i> (27); Bisection (31)	
	Mathematical Organization and Metalanguage	32
	The Standard Format of Problems (32); Standard Names and Standard Representation (33); Structuration (37); Recording (39)	
	The “Conformal Translation”	40
	Table 1: Akkadian Terms and Logograms with Appurtenant Standard Translation. (43); Table 2: The Standard Translations with Akkadian and Logographic Equivalents (47)	
III	SELECT TEXTUAL EXAMPLES	50
	BM 13901 #1	50
	BM 13901 #2	52
	BM 13901 #3	53
	YBC 6967	55
	BM 13901 #10	58
	BM 15285 #24	60

	VAT 8390 #1	61
	YBC 6295	65
	BM 13901 #8–9	66
	BM 13901 #12	71
	BM 13901 #14	73
	VAT 8389 #1	77
	VAT 8391 #3	82
	TMS XVI	85
	TMS IX	89
IV	METHODS	96
	“Naive” Cut-and-Paste Geometry	96
	Scaling and Other Changes of Variable	99
	Accounting, Coefficients, Contributions	100
	Single (and Other) False Positions – and Bundling	101
	Drawings? Manifest or Mental Geometry?	103
V	FURTHER “ALGEBRAIC” TEXTS	108
	BM 13901 #18	108
	YBC 4714	111
	#1–#3 (132); #4–7, 10–12 (133); #8–9 (133); #13–20 (133); #21–28 (134); #29 (134); #30–39 (135); General Commentary (136)	
	BM 85200 + VAT 6599	137
	The Third-Degree Problems (149); The second degree: Length–Width, Depth–Width, and Length–Depth (154); Second-Degree <i>igûm-igibûm</i> -Problems (158); First-Degree Problems (159); Clues to Teaching Methods (161)	
	AO 8862 #1–4	162
	#1 (169); #2 (170); #3 (171); Average and Deviation (172); #4 (174)	
	YBC 6504	174
	AO 6770 #1	179
	TMS VII	181
	#1 (185); #2 (186); A Concluding General Observation (188)	
	TMS VIII	188
	#1 (191); #2 (193)	
	TMS XIX	194
	#1 (197); #2 (197)	
	YBC 4668, Sequence C, #34, #38–53	200
	YBC 4713 #1–8	” 3 0 – 9 8 4 7

VI	QUASI-ALGEBRAIC GEOMETRY	227
	Introductory Considerations	227
	Angles and Similarity (227); Perpendicularity and Orientation (228); Rectangles, Triangles, Trapezia, and “Surveyors’ Formula” (229)	
	IM 55357	231
	VAT 8512	234
	Str 367	239
	YBC 4675	244
	UET V, 864	250
	YBC 8633	254
	Db ₂ -146	257
	YBC 7289	261
	TMS I	265
	VAT 6598 #6–7	268
	BM 85194 #20–21	272
	BM 85196 #9	275
	Summary Observations	276
VII	OLD BABYLONIAN “ALGEBRA”: A GLOBAL CHARACTERIZATION	278
	Algebra?	278
	Equations (282)	
	Distinctive Characteristics	282
	The Given and the Merely Known (283); “Pedantic Repetitiveness” (284); Favourite Configurations (285); Favourite Problems (286); “Remarkable Numbers” (287); “Broad Lines” and “Thick Surfaces” (291)	
	Did They “Know” It?	292
	Zero (293); Negative Numbers (294); Irrational Numbers? (297); Logograms as “Mathematical Symbols”? (298)	
	Overall Organization	299
	Technical Terminology (299); Mathematics? (302)	
VIII	THE HISTORICAL FRAMEWORK	309
	Landscape and Periodization	309
	Scribes, Administration – and Mathematics	311
IX	THE “FINER STRUCTURE” OF THE OLD BABYLONIAN CORPUS	317
	Description of the Groups	319
	Group 7: The Eshnunna Texts (319); Group 8: The Susa Texts (326); Groups 6 and 5: Goetze’s “Northern” Groups (329); Groups 4 and 3: Goetze’s “Uruk Groups” (333); Group 1: The “Larsa” Group (337); Group 2 – a Non-Group? (345); The Series Texts (349); Old Babylonian Ur and Nippur (352); Summarizing (358)	
	The Outcome	358

X	THE ORIGIN AND TRANSFORMATIONS OF OLD BABYLONIAN ALGEBRA	362
	Practitioners' Knowledge and Specialists' Riddles	362
	A Long and Widely Branched Tradition: the Lay Surveyors	368
	The Sumerian School: the Vocabulary as Evidence	375
	The "Surveyors' Proto-algebra"	378
	Scholastization	380
	An Aside on the Pythagorean Rule	385
	The Later Phases	387
	Seleucid Procedure Texts	389
	BM 34568	391
XI	REPERCUSSIONS AND INFLUENCES	400
	Greek Theoretical Mathematics	400
	Demotic Egypt	405
	Greek Underbrush	406
	India	408
	Impact in Islamic and Post-Islamic Mathematics: Towards Early Modern Algebra	410
	ABBREVIATIONS AND BIBLIOGRAPHY	418
	INDEX OF TABLETS	428
	INDEX OF AKKADIAN AND SUMERIAN TERMS AND KEY PHRASES	434
	NAME INDEX	440
	SUBJECT INDEX	443