Mathematics, Mesopotamian

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Mesopotamian mathematics and writing began together as book-keeping tools in proto-literate Uruk (c. 3300 BCE), apparently key constituents in the legitimization of the transformation of a redistributive into a statal social system. At least until 2000 BCE, accounting and mathematically determined allocation of rations and land seem to have carried connotations of "social justice". In the second and first millennium, mathematics still served accounting and planning purposes, and in the later first millennium in a sophisticated mathematical astronomy connected to astrology, but no textual evidence suggests a continued ideological role.

Beyond texts *using* mathematics we have "properly mathematical texts", texts serving mathematics education. The protoliterate school texts are all "model documents", distinguishable from administrative documents solely by the absence of a seal and the presence of round or large numbers. As scribes turn up as a separate *profession* in Šuruppak around 2600 BCE, we find the beginning of "supra-utilitarian" mathematics: mathematics which deals with *topics* belonging to scribal practice (ration distribution, mensuration, etc.) but whose sophisticated *problems* would never turn up in practice. They are also found in the Sargonic school, whereas Ur III, in spite of its extensive reliance on refined mathematical accounting, offers nothing but model documents. Supra-utilitarian mathematics returns powerfully in the Old Babylonian school, 1800–1600 BCE, in particular in the shape of an "algebra" based on measurable lines and surfaces (see BABYLONIAN GEOMETRY). From 1600 to 400 only a very modest mathematical corpus is known (accounting, surveying, a few tables). After 400 and through the Seleucid era, the environment of scholar-scribes takes some renewed interest in mathematics, including supra-utilitarian mathematics, but by then mathematical administration and the teaching of common mathematical practitioners was probably made in Aramaic on ephemeral writing media. Specialists belonging to the same environment produced the mathematical astronomy.

Both practical and supra-utilitarian texts always aim at numerical results. Explicit demonstration and the formulation of "theorems" was never a Babylonian concern, but not least Old Babylonian texts show clear evidence of genuine understanding; a few texts also contain clear didactical *explanations* of the meaning of operations.

The number system of the proto-literate period is sexagesimal – or rather sexal-decimal (as Roman numerals are dual-quintal), with units 1, 10, 60, 600, 3600, 36000. For specific purposes a system with units 1, 10, 60, 120, 1200, 7200 was used. A number of metrological sequences (for length, area, hollow measure, time) seem to represent mathematical regularizations of older natural measures. Over the third millennium, metrologies were gradually adapted both to a common factor 60 and so as to fit bureaucratic procedures – aims sometimes in conflict.

Ur III administration solved the conflict by introducing a *place value system* (still with base 60) supported by tables of technical constants and reciprocals and for

multiplication and for converting non-sexagesimal metrologies. This system, which did not indicate absolute order of magnitude (having no “sexagesimal point”) was used exclusively in intermediate calculations (and later in school texts and in mathematical astronomy). For other purposes, the old system served, in the second and first millennia in combination with the Akkadian number words for 100 and 1000.

Suggestions for further reading:

Jens Høyrup, “Mathematics and Early State Formation, or, the Janus Face of Early Mesopotamian Mathematics: Bureaucratic Tool and Expression of Scribal Professional Autonomy”, pp. 45–87, 296–306 *in* Jens Høyrup. *In Measure, Number, and Weight. Studies in Mathematics and Culture.* New York: State University of New York Press, 1994.

Jens Høyrup, *Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin*. (Studies and Sources in the History of Mathematics and Physical Sciences). New York: Springer, 2002.

Hans J. Nissen, Peter Damerow & Robert Englund, *Archaic Bookkeeping: Writing and Techniques of Economic Administration in the Ancient Near East*. Chicago: Chicago University Press, 1993.

Eleanor Robson, *Mesopotamian Mathematics 2100–1600 BC. Technical Constants in Bureaucracy and Education*. (Oxford Editions of Cuneiform Texts, 14). Oxford: Clarendon Press, 1999.

Eleanor Robson, *Mathematics in Ancient Iraq: A Social History*. Princeton & Oxford: Princeton University Press, 2008.