

Fibonacci, Leonardo

Summary:

Leonardo Fibonacci (*1170–80, † after 1241) is known for mathematical writing, often submitting topics from the practical traditions to the approach of scholarly mathematics: *Liber abbaci*, *Pratica geometriae*, *Flos*, *Letter to master Theodorus*, *Liber quadratorum*. He makes use of Greek and Arabic sources translated into Latin as well as unidentified scholarly and “vernacular” Arabic, Byzantine and Ibero-Provençal material. His influence was probably much more modest than has been believed.

Leonardo Fibonacci (probably born between 1170 and 1180, died after 1242 CE) was one of the transmitters of Arabic mathematical knowledge to Christian Europe, and the only one known by name who transmitted matters inspired by practical and commercial arithmetic. He was born in Pisa, a son of Guglielmo *de filiis Bonaccii*, whence the modern surname Fibonacci. Outside Pisa he referred to himself as *pisano*, while two documents from Pisa and several of his introductions identify him as *bigollo*, probably meaning “the traveller” (Bonaini; Ulivi, 247–254). In boyhood he was brought to Bejaïa (Bougie, Bijāya) by his father, who served there as “public scribe for the Pisa merchants”, so that “for some days” he might learn practical calculation – thus the preface to his *Liber abbaci* (on which below). Having become captivated by the Hindu-Arabic-numerals, the account continues, he studied (what was done with) them on business travels to Egypt, Syria, Constantinople, Sicily and Provence (*Scritti* I, 1). He was still active in 1241 CE, after which we hear nothing about him.

In 1202, he wrote a first version of the *Liber abbaci*, which has been lost; its title does not refer to an abacus of any kind but apparently means “Book of practical computation” (unless it intends to render Arabic *Kitāb al-mu‘āmalāt*). Around a dozen complete and incomplete copies exist of a second version from 1228, in which Fibonacci “added certain necessary and eliminated certain superfluous matters” (*Scritti* I,1). Another lost work to which the *Liber abbaci* refers once and which an anonymous mid-fifteenth-century writer (Firenze, Biblioteca Nazionale, Palat. 573, fol. 433^v) knew by name is *Liber minoris guise*, “Book in a smaller manner” introducing to commercial arithmetic. Other works are *Pratica geometriae* (1220), *Flos*, a *Letter to Master Theodorus*, and *Liber quadratorum* – all in (*Scritti* II). *Liber quadratorum* is dated 1225, but claims to have

been presented to the Emperor Frederick II von Hohenstaufen (“Holy Roman Emperor” but also King of Sicily) during his visit to Pisa, which took place in 1226; the others are undated but probably close in date. The revised *Liber abbaci* is dedicated to Michael Scot, formerly a translator based in Toledo but now philosopher at Frederick II’s Sicilian court. Some of the dedicatees of the other works (perhaps all of them) were also connected to Emperor Frederick’s court; Fibonacci himself did not belong to the Emperor’s circle.

The *Liber abbaci* starts by describing the Hindu-Arabic numerals and how to compute with them. These were known in Christian Europe since the Latin translations of the twelfth century; they are also used in late twelfth-century notarial documents from Perugia (Burnett V, 254). However, what Fibonacci describes and uses in all his extant works is clearly derived, not from Latin or Italian forerunners but from Maghreb habits, including notations for composite fractions such as $\frac{1}{2} \frac{5}{8} \frac{3}{11}$ (meaning $\frac{3}{11} + \frac{5}{8}$ of $\frac{1}{11} + \frac{1}{2}$ of $\frac{1}{8}$ of $\frac{1}{11}$); he also writes mixed numbers with the fraction to the left, $\frac{2}{5}4$ where we would write $4\frac{2}{5}$. The bulk of the book presents commercial arithmetic, including complicated “recreational” problems (spoken of as “rarities” – *tarāʿif* or *nawādir* – in Arabic mathematics; mathematical puzzles about three men finding a purse and dividing its contents according to a complicated prescription, etc.). Often, however, as Fibonacci says, he deals with the matter *magistraliter* (“in the manner of the schools”), distancing itself from “vernacular” ways (*vulgi modus*). The *Liber abbaci* thus presents an attempt to submit its subject-matter to the norms of scholarly, not least Euclidean mathematics. Chapter 14 deals with roots and operations with bi- and trinomials involving radicals, with reference to Euclid’s *Elements* II and X; chapter 15 mainly treats of algebra, in a style going back to al-Khwārizmī (first half of the 3rd/9th c.) and Abū Kāmil Shujāʿ al-Misrī (late 3rd/9th or early 4th/10th c.) but also using *Elements* II and proportion theory where these would apply purely algebraic arguments.

The *Pratica geometriae* represents a similar integration of levels, looking to theory rather than practice (*Scritti* I, 1). It introduces matters borrowed from the Arabic tradition of *ʿilm al-misāha* (the “science of measurement”, practical plane and solid geometry); the metrology used in Pisa; instructions for surveying sloping surfaces; basic trigonometry; and the extraction of square and cube roots – all discussed within a Euclidean framework, and together with classical problems like the doubling of the cube and partition of figures.

The *Flos* solves difficult problems that are all algebraic from a modern point of view but had no such connection at the time. First it shows that a particular cubic equation can have no solution within the domain of Euclidean irrationals

and states an approximate root (not disclosing how it was found); next it deals with intricate recreational problems (of types “purchase of a horse” etc.) that translate into indeterminate linear problems in several variables; in one of them Fibonacci makes use of two unknowns, *causa* (pseudo-Latinization of contemporary Iberian or Italian for “thing”) and *res* (Latin with the same meaning); another problem works with three unknowns, *res*, *dragma* and *bursa* (the unknown contents of a “purse”) (*Scritti* II, 236, 238).

The *Letter to Master Theodorus* (another philosopher at Frederick II’s Palermo court) takes up similar matters. *Liber quadratorum* explores the problem to find three square numbers such that (in modern terminology) $x^2 - y^2 = y^2 - z^2 = 5$, which leads Fibonacci to investigate more general properties of such “congruent numbers” (Oort).

Euclid, as mentioned, was a very important authority for Fibonacci. Sometimes he uses the Greco-Latin translation of the *Elements*, sometimes he quotes from memory, apparently from an Arabic or an Arabo-Latin version (Folkerts, IX). He cites and uses Ptolemy, Menelaus and Theodosius, and uses the Latin translations of the 9th-10th-century astronomer-mathematicians Banu Mūsā and of Ahmad ibn Yūsuf, citing only the latter by name.

He also knew and drew verbatim upon Gerard of Cremona’s translations of al-Khwārizmī’s *Algebra* and of a *Liber mensurationum* (“Book on Mensuration”) attributed to an otherwise unidentified Abū Bakr “called Heus”. Material borrowed from Abū Kāmil and al-Karājī is taken from memory or indirectly. His explanation of *algebra et almuchabala* (Ar. *al-jabr wa’l-muqābala*, “completion and balancing”) as “proportion and restoration” (*Scritti* I, 406) raises doubts about the depth of his familiarity with Arabic, but he must have had access to Arabic knowledge that was not diffused in the Latin-Christian world. His familiarity with practical arithmetic beyond numerals and computation was acquired at least in part in Constantinople and in the Ibero-Provençal area. In the latter area he is also likely to have learned about 12th-century investigations of theoretical arithmetic which (like other late developments in al-Andalus) have not been transmitted in Arabic.

Fibonacci’s influence in Italy has probably been overrated. The early Italian “abbacus school” (a school type emerging in the later 13th century and teaching a two-year course in practical arithmetic for the sons of merchants and artisans) appears to have been primarily inspired by the same “vernacular” practices Fibonacci had cited. One school tradition in Florence, from Paolo dell’Abbaco (d. c. 1367) to Benedetto of Florence (d. 1479), held Fibonacci in high honour, but made its own work within the normal abbacus tradition.

Very few medieval university scholars knew his work: Fibonacci's contemporary Jordanus de Nemore perhaps did so, Jean de Murs a hundred years later certainly did.

Bibliography:

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