Constrained Horn Clause Verification

Bishoksan Kafle  John Gallagher
Roskilde University

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Overview

Motivation

Imperative language to CHCs

Tools, Techniques

Our approach to CHC verification

Conclusion and Future works
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Why Constrained Horn Clause (CHC) verification?

CHC is a

• suitable intermediate language to express system’s behavior
• suitable target language for translating a variety of
  ○ languages i.e. imperative, functional, concurrent etc.
  ○ computational models e.g. state machines, transition systems, Markov chain etc.
• a large number of research community working on this including Microsoft.
• success story : Windows device driver verification etc.
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Translation to CLP form

- **semantics based** translation (systematic)
- based on **partial evaluation** of imperative language’s interpreter (e.g. XC)

**Imperative Program**

```plaintext
i=0; a=0; b=0;
assume(n > 0);
while (i < n){
    if (*){
        a=a+1;
        b=b+2;
    }else{
        a=a+2;
        b=b+1;
    }
i++; }
assert(a+b == 3*n);
```

**CLP Program**

```plaintext
false:- N>0,I=0,A=0,B=0, l(I,A,B,N).

l(I,A,B,N):- I < N, l_body(A,B,A1,B1),
      I1 = I+1, l(I1,A1,B1,N).
l(I,A,B,N):- I >=N, A + B > 3 * N.
l(I,A,B,N):- I >=N, A + B < 3 * N.
l_body(A0,B0,A1,B1):- A1 = A0+1,
                  B1 = B0+2.
l_body(A0,B0,A1,B1):- A1 = A0+2,
                  B1 = B0+1.
```

Whole-Systems Energy Transparency
Definitions

**Constrained Horn Clause (CHC)**

A predicate logic formula, \( H(X) \leftarrow \phi \land B_1(X_1), \ldots, B_k(X_k) \) where \( \phi \) is a conjunction of constraints with respect to some background theory, \( X_i, X \) are (possibly empty) vectors of distinct variables, \( B_1, \ldots, B_k, H \) are predicate symbols, \( H(X) \) is the head of the clause and \( \phi \land B_1(X_1) \land \ldots \land B_k(X_k) \) is the body.

**Integrity constraints**

\[ \text{false} \leftarrow \phi \land B_1(X_1), \ldots, B_k(X_k). \]

where \text{false} is always interpreted as false.

CHC is a software verification community’s terminology for CLP
From now on CHC and CLP are used interchangeably
CHC Verification

**CHC verification problem**

- given a set of CHCs $P$,
- is to check whether there exists a model of $P$
- $P$ has a model if and only if $P \not\models \text{false}$.

**Representation of Interpretations**

- An interpretation of $P$: a set of *constrained facts* of the form $A \leftarrow C$, where
- $A$ is an atomic formula $p(Z_1, \ldots, Z_n)$ where $Z_1, \ldots, Z_n$ are distinct variables, and
- $C$ is a constraint over $Z_1, \ldots, Z_n$. 
Models

Minimal models

- A model of $P$ is an interpretation that satisfies each clause.
- There exists a minimal model with respect to the subset ordering, denoted $M[P]$,
- the minimal model $M[P]$ is equivalent to the set of atomic consequences of $P$ (model vs. proof)
- $P \models p(v_1, \ldots, v_n)$ if and only if $p(v_1, \ldots, v_n) \in M[P]$
- $M[P]$ can be computed as the least fixed point (lfp) of an immediate consequences operator, $T_P^C$
Proofs

Proof by over-approximation of the minimal model

- It is sufficient to find a set of constrained facts $M'$ such that $M[P] \subseteq M'$, where false $\not\in M'$.

Proof by specialisation

- A specialisation of $P$ with respect to an atom $A$ is the transformation of $P$ to another set of CHCs $P'$ such that $P \models A$ if and only if $P' \models A$.
- can be viewed as program optimization
- In our context, w.r.t. to the atom false
Analysis

Convex polyhedron (hull) approximation (CHA)

- CHA is a program analysis technique based on abstract interpretation.
- When applied to $P$ it constructs an over-approximation $M'$ of the minimal model of $P$, where $M'$ contains at most one constrained fact $p(X) \leftarrow C$ for each predicate $p$.
- where the constraint $C$ is a conjunction of linear inequalities, representing a convex polyhedron.
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Tools and Techniques

- CHC verification has gained interests from CLP and software verification communities
- Several techniques such as Abstract Interpretation (AI), Counter Example Guided Abstraction Refinement (CEGAR) and program specialization have been proposed in the literature to solve CHC program.
- Tools: Z3, QARMC (CEGAR), TRACER, VeriMAP (specialisation) etc.
Pitfall

- Several challenging problems, no single technique is powerful enough
- but some of these techniques perform better in some cases while others in some other cases,
- that is, they usually miss the aspect of the other.
- So their combination could give a better result?

Our approach is to combine the strength of techniques developed for CLP and Software Verification in the same framework
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Summary of our approach

**Figure:** Tool chain overview (CHC verification).
Approach

- abstract interpretation over convex polyhedra domain (main engine),
  - loses precision due to merge and widening operators
  - CLP transformation techniques such as unfolding, predicate splitting, specialization could make analysis results better
- specialisation of the constraints in CHCs using abstract interpretation of query answer transformed clauses (simulates the computation tree semantics of CLP), and
- refinement by predicates splitting (guided by abstract trace)
- generation of new program based on extended set of constraints (CEGAR simulation)
Running Example

Original CHC P

false :- 1*A>0, 1*B=0, 1*C=0, 1*D=0, l(B,C,D,A).

l(A,B,C,D) :- -1*A+1*D>0, 1*A+ -1*G= -1,
    l_body(B,C,E,F), l(G,E,F,D).

l(A,B,C,D) :- 1*A+ -1*D>=0, 1*B+1*C+ -3*D>0.

l(A,B,C,D) :- 1*A+ -1*D>=0, -1*B+ -1*C+3*D>0.


l_body(A,B,C,D) :- 1*A+ -1*C= -2,1*B+ -1*D= -1.

Specialised CHC Sp(P)

c1. false :- 1*A>0, 1*B=0, 1*C=0, 1*D=0, l(B,C,D,A).

c2. l(A,B,C,D) :- 2*A+ -1*B>=0, -1*A+1*D>0, -1*A+1*B>=0,
    3*A+ -1*B+ -1*C=0, 1*A+ -1*E= -1,
    l_body(B,C,F,G), l(E,F,G,D).

c3. l(A,B,C,D) :- 3*A+ -3*D>0, 1*D>0,
    2*A+ -1*B>=0, -3*A+3*D> -3,
    -1*A+1*B>=0, 3*A+ -1*B+ -1*C=0.

c4. l_body(A,B,C,D) :- -1*A+2*B>=0, 2*A+ -1*B>=0,
    1*A+ -1*C= -1,1*B+ -1*D= -2.

c5. l_body(A,B,C,D) :- -1*A+2*B>=0, 2*A+ -1*B>=0,
    1*A+ -1*C= -2,1*B+ -1*D= -1.

Constraint facts for QA(P)

l_body(A,B,C,D) :- -1*A+2*B>=0,2*A+ -1*B>=0.

l(A,B,C,D) :- 2*B+ -1*C>=0,1*D>0,-1*B+2*C>=0,-1*B+ -1*C+3*D> -3,3*A+ -1*B+ -1*C=0.
false :- true.
false :- true.
l(A,B,C,D) :- true.
l(A,B,C,D) :- true.
CHA Analysis

CHA Result on Sp(P)

\begin{align*}
l_{\text{body}}(A,B,C,D) & :- 1B + -1D \geq -2, -1B + 1D = 1, -1A + 2B = 0, \\ & 2A + -1B = 0, 1A + 1B + -1C + -1D = -3. \\
\text{false} & :- \text{true}. \\
l(A,B,C,D) & :- 1D > 0, 2A + -1B \geq 0, -1A + 1B = 0, -3A + 3D > -3, \\ & 3A + -1B + -1C = 0.
\end{align*}

• presence of constrained fact for \textit{false} $\rightarrow P$ not safe
• CHA returns counter example trace c1(c3) in the form of trace term
• check trace for feasibility by collecting constraints from the clauses,
  ○ if feasible then our analysis terminates and returns bug
  ○ else refine Sp(P)
Tool chain overview

Figure: Tool chain overview (CHC verification).
Counter example analysis

Interpolant

Given two constraints $C_1, C_2$ such that $C_1 \land C_2$ is unsatisfiable, an interpolant is a constraint $I$ with (i) $C_1 \rightarrow I$, (ii) $I \land C_2$ is unsatisfiable and (iii) $I$ contains variables common to both $C_1$ and $C_2$.

Predicate splitting

Let $I(X)$ be such a constraint over set of variables $X$, and $p(X) \leftarrow c(X)$ a constrained fact, then we split this fact into $p(X) \leftarrow c(X), I(X)$ and $p(X) \leftarrow c(X), \neg I(X)$. 
Predicate splitting

- \( I(A,B,C,D) = A+ -3*B+C+D=<0 \) is the interpolant computed from the trace \( c1(c3) \) for predicate \( l(A,B,C,D) \).
- splitting \( l(A,B,C,D) :- 1*D>0,2*A+ -1*B>=0,-1*A+1*B>=0,-3*A+3*D> -3, 3*A+ -1*B+ -1*C=0. \) with the interpolant produces (after constraint simplification)

\[
l(A,B,C,D) :- -4*A+4*B+ -1*D>=0,1*D>0,-3*A+3*D> -3, 2*A+ -1*B>=0, 3*A+ -1*B+ -1*C=0.
\]

- Based on these extended set of constraint facts and \( Sp(P) \) we generate a new CHC through specialization which is more precise than the original problem.
- In the next iteration we are able to show the presence of a bug and thus our procedure terminates.
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Conclusion:

- presented an approach for CHC verification based on constraint propagation by specialization, program transformation, convex hull analysis and property-based specialisation that splits predicates, leading in turn to more precise convex polyhedral analyses
- experimental results on some challenging benchmark problems from software verification repository prove the feasibility of our approach

For the future:

- explore new ways of refining polyhedra abstraction
- understand better the connection between program specialization and CEGAR
- interface with SMT solvers (for satisfiability checking w.r.t. to some background theory and interpolants generation)
Thanks for your attention!

Questions?