Resource analysis of a C program by analyzing its Horn clause representation

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Resource analysis can answer…

```
main( uint c,  uint d){
    int a=c,b=d;
    while (a>0){
     if (*)
       b++;
      else
       while (b>0)
           b--;
      a--; }
     return 0;
 }
```

- Amount of resource consumed by this program
- Part of code responsible for the highest amount of resource consumption
- Resource consumed by the inner loop etc.

- Resource = time, energy, memory etc.

Example

```
 main( uint c,  uint d){
0.    int a=c,b=d;
1.    while (a>0){
2.     if (*)
3.        b++;
        else
4.        while (b>0)
5.            b--;
6.      a--; }
7.     return 0;
 }
```

- Challenging problem: for the state of the art resource analysis tools (CiaoPP, PUBS etc.)
- The counter for the inner loop is conditionally increased by the outer one
- Path sensitive reasoning is necessary to derive a tight bound

Bound

```
main( uint c,  uint d){
    int a=c,b=d;
    while (a>0){
     if (*)
       b++;
      else
       while (b>0)
           b--;
      a--; }
     return 0;
 }
```

- Outer loop: can be executed at most c times
- The counter for the inner loop can be incremented by outer at most c times
- So inner loop can be executed at most c+d times

Bound=c+d

How to derive this linear bound automatically?

What information is necessary?

```
main( uint c,  uint d){
    int a=c,b=d;
    while (a>0){
     if (*)
        b++;
       else
        while (b>0)
            b--;
       a--; }
     return 0;
 }
```

- ranking functions of the loops
- The effects of the outer loop (increment or decrement) on the ranking function of the inner one (invariants)
- Invariants of the program

Analysis

- We base our analysis on some intermediate representation called Horn clauses rather than on the source language
- The use Horn clauses allows reuse of the Horn clause tools (e.g., to find invariants);
- The use of Horn clause representation makes it easier to compute ranking functions using off the shelf tool (e.g., PPL)

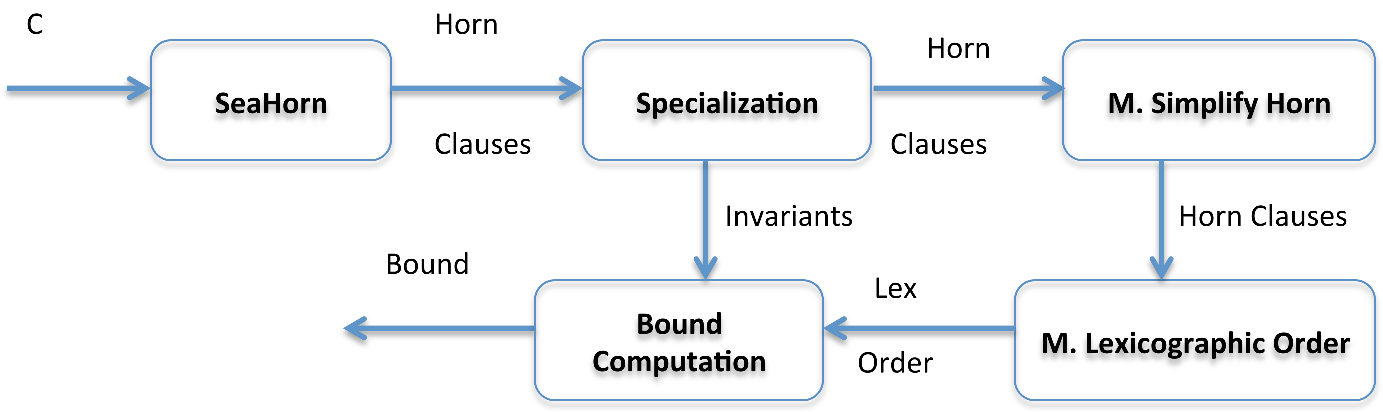
Our approach

Program invariants

Ranking functions

Bound computation

Architecture of our tool-kit



[1] Moritz Sinn, Florian Zuleger, Helmut Veith:
A Simple and Scalable Static Analysis for Bound Analysis and Amortized Complexity Analysis. CAV 2014

Horn clause

$$p(X) \leftarrow \phi \wedge p_1(X_1) \wedge \ldots \wedge p_k(X_k)$$

Linear clause (Transition system)

$$p(X') \leftarrow \phi \wedge q(X)$$

$$p(X') \leftarrow \phi$$

Restricting the shape of constraints

$$p(X') \leftarrow X' \leq X + K \wedge q(X), K \in \mathbb{Z}^n$$

Horn clauses

Restricting the shape of constraints

$$p(X') \leftarrow X' \leq X + K \wedge q(X), K \in \mathbb{Z}^n$$

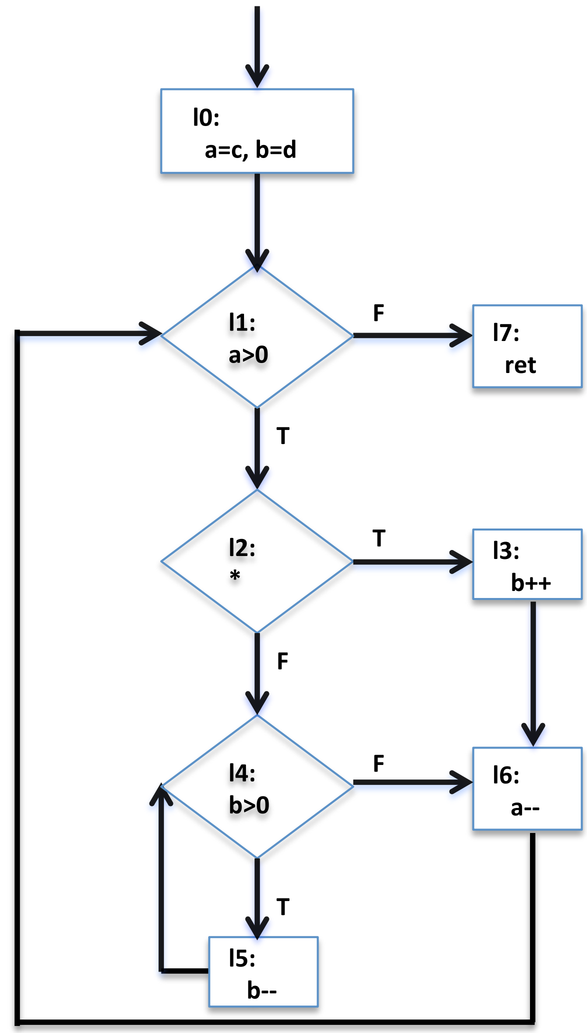
Allowing parameters

$$p(X') \leftarrow X' \leq X + K \wedge q(X), k_i \in \mathbb{Z} \cup Params$$

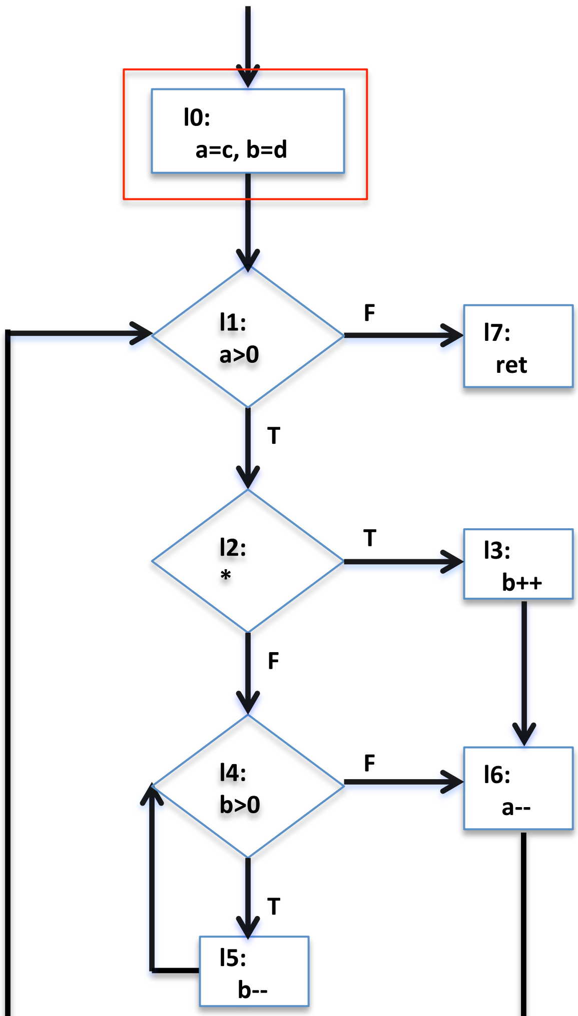
Params are program's input params eg. c and d

Horn clause generation (1)

```
 main( uint c,  uint d){
0.    int a=c,b=d;
1.    while (a>0){
2.     if (*)
3.        b++;
       else
4.        while (b>0)
5.            b--;
6.       a--; }
7.     return 0;
 }
```

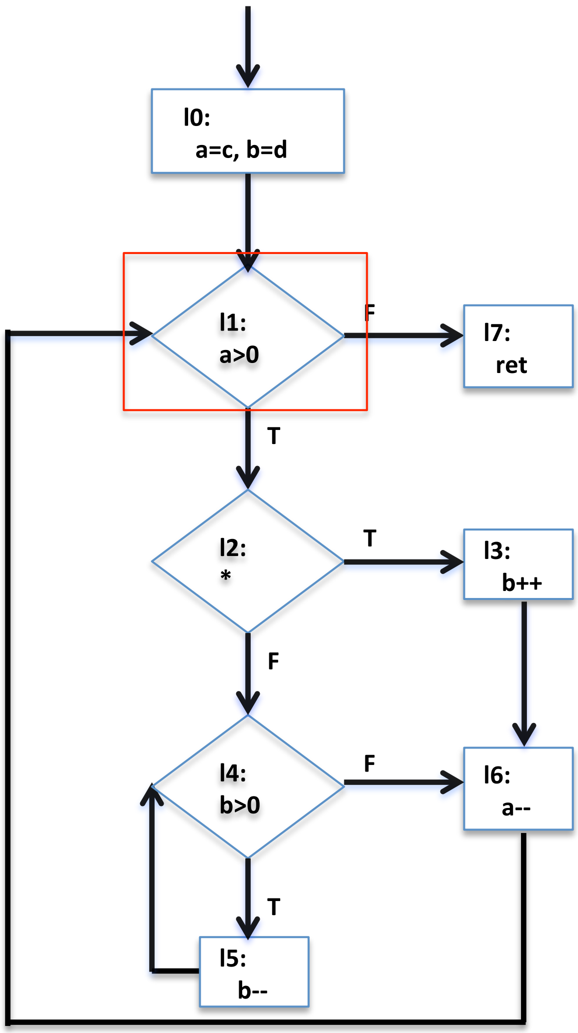


Horn clauses Generation (2)




```
p0(A,B,C,D) ← true.
p1(A,B,C,D) ← A=C,B=D, C>=0,D>=0,
              p0(A,B,C,D).
p1(A,B,C,D) ← A=A1-1,p6(A1,B,C,D).
p2(A,B,C,D) ← A>0,p1(A,B,C,D).
p3(A,B,C,D) ← p2(A,B,C,D).
p4(A,B,C,D) ← p2(A,B,C,D).
p4(A,B,C,D) ← B=B1-1,p5(A,B1,C,D).
p5(A,B,C,D) ← B>0,p4(A,B,C,D).
p6(A,B,C,D) ← B=<0,p4(A,B,C,D).
p6(A,B,C,D) ← B=B1+1,p3(A,B1,C,D).
p7(A,B,C,D) ← A=<0,p1(A,B,C,D).
ret         ← p7(A,B,C,D).
```

Horn clauses Generation




```
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               p0(A,B,C,D).
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p2(A,B,C,D) ←  A>0,p1(A,B,C,D).
p3(A,B,C,D) ←  p2(A,B,C,D).
p4(A,B,C,D) ←  p2(A,B,C,D).
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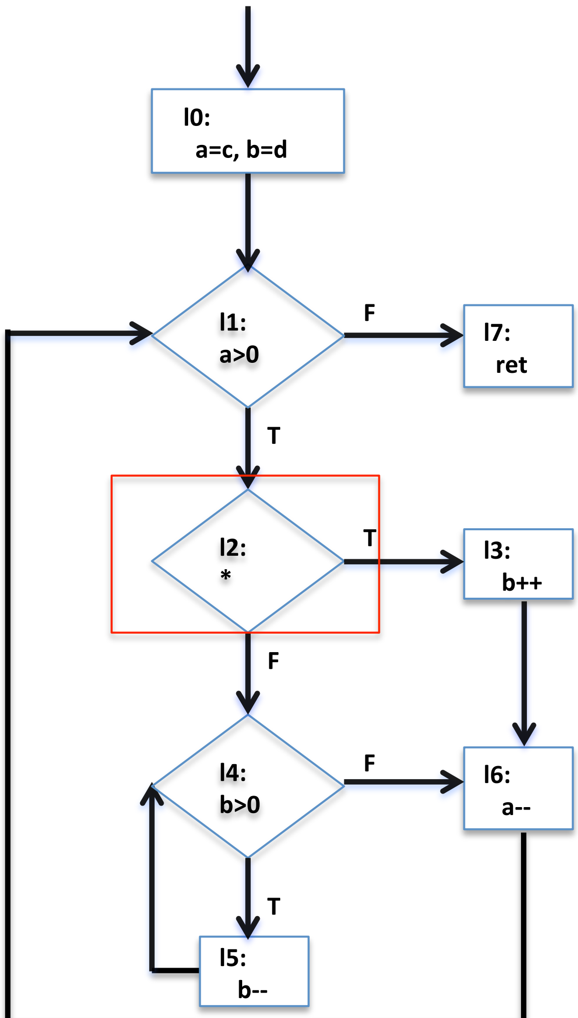
Horn clauses Generation

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p0(A,B,C,D) ←  true.
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p1(A,B,C,D) ←  A=A1-1,p6(A1,B,C,D).
p2(A,B,C,D) ←  A>0,p1(A,B,C,D).
p3(A,B,C,D) ←  p2(A,B,C,D).
p4(A,B,C,D) ←  p2(A,B,C,D).
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p7(A,B,C,D) ←  A=<0,p1(A,B,C,D).
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```
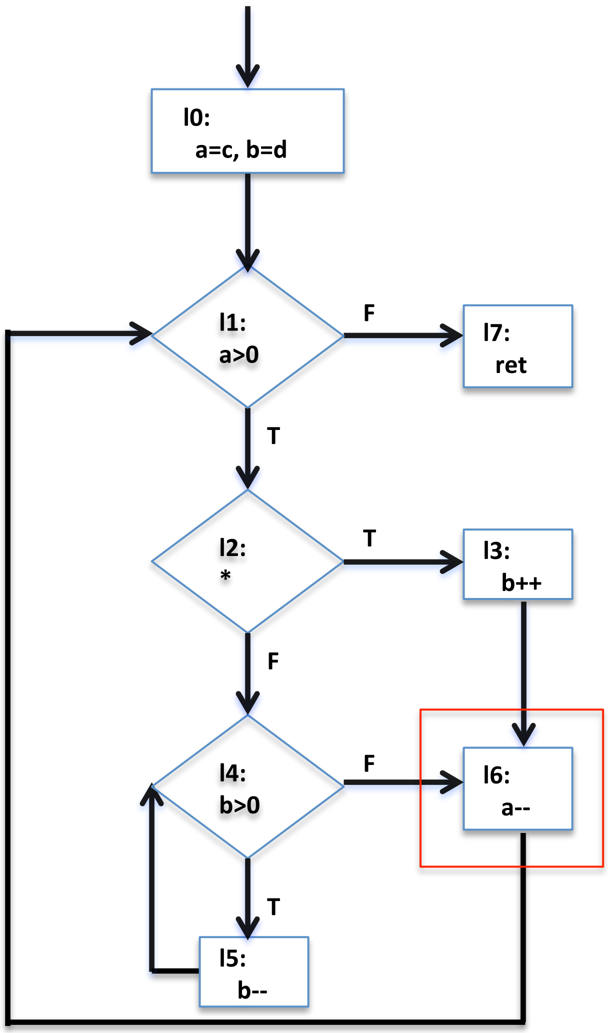

Horn clauses Generation




```
p0(A,B,C,D) ←  true.
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ret         ←  p7(A,B,C,D).
```
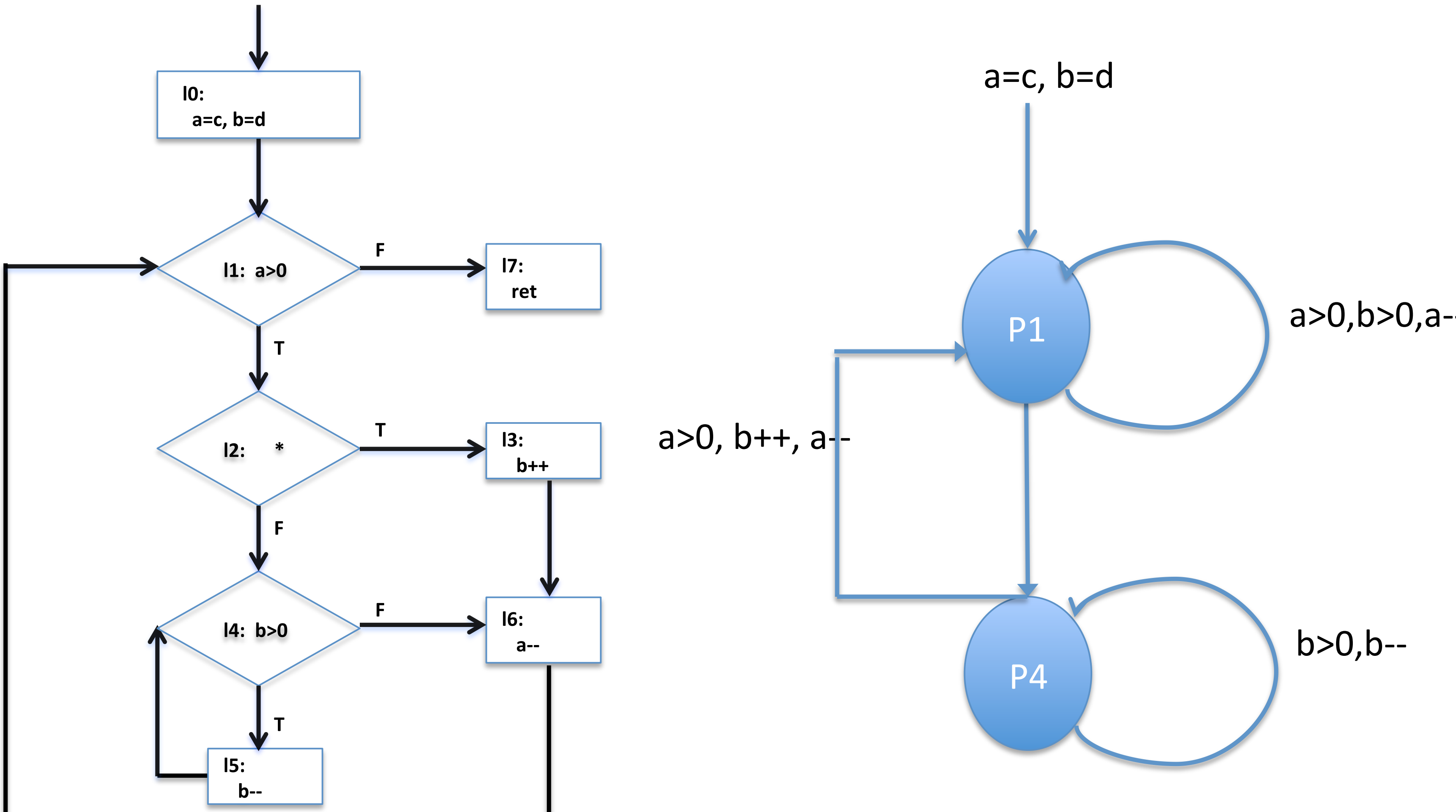
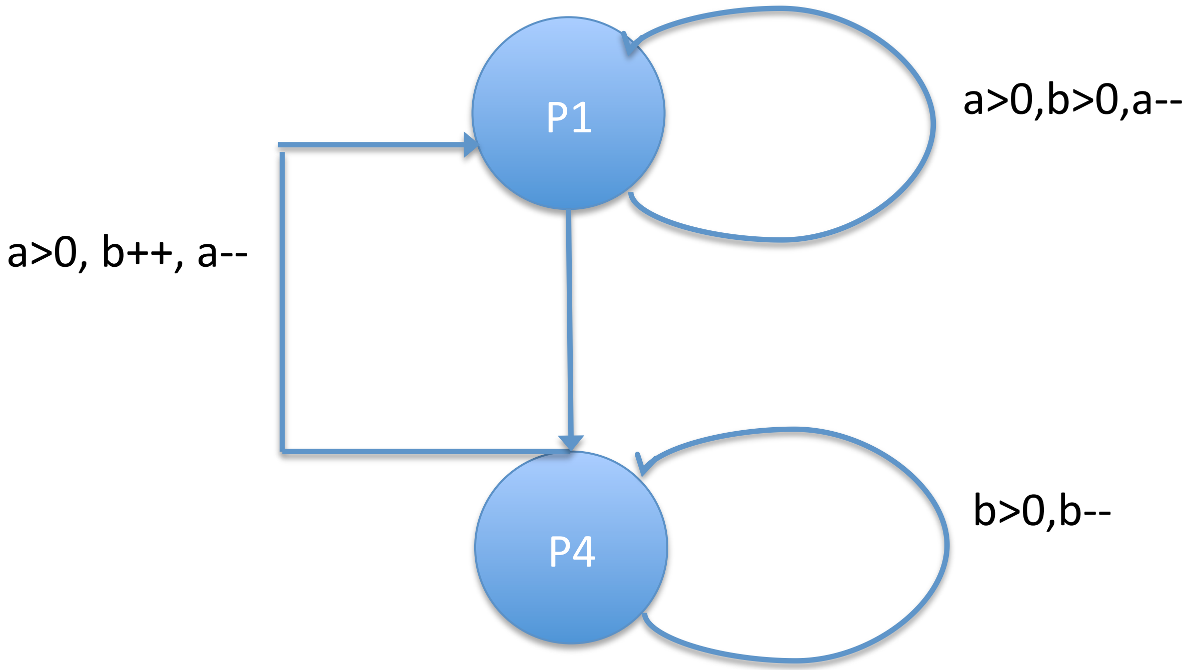

Clauses can be viewed as set of equations!

Single path linear constraint loops



Special form of Horn clauses (Abstraction of loops)




```
p1(A,B,C,D) ←  A=<E-1,B=<F+1,p1(E,F,C,D)
p1(A,B,C,D) ←  A=<E-1,B=<F,p1(E,F,C,D)
p4(A,B,C,D) ←  A=<E,B=<F-1,p4(E,F,C,D)
```

Lexicographic ranking function

```
T1: p1(A,B,C,D) ←  A=<E-1,B=<F+1,p1(E,F,C,D)
T2: p1(A,B,C,D) ←  A=<E-1,B=<F,p1(E,F,C,D)
T3: p4(A,B,C,D) ←  A=<E,B=<F-1,p4(E,F,C,D)
```

- <A,A,B>
- Either A is decreasing; or
- B is decreasing and A is not increasing

Bound computation

```
T1: p1(A,B,C,D) ← A=<E-1,B=<F+1,p1(E,F,C,D)
T2: p1(A,B,C,D) ← A=<E-1,B=<F,p1(E,F,C,D)
T3: p4(A,B,C,D) ← A=<E,B=<F-1,p4(E,F,C,D)
```

InitVal: A=c, B=d

Lex: <A,A,B>

- Bound(T1)= InitVal (A)=c
- Bound(T2)= InitVal (A) + Bound(T1)*increment(A,T1) =c+0=c

Bound computation

```
T1: p1(A,B,C,D) ←  A=<E-1,B=<F+1,p1(E,F,C,D)
T2: p1(A,B,C,D) ←  A=<E-1,B=<F,p1(E,F,C,D)
T3: p4(A,B,C,D) ←  A=<E,B=<F-1,p4(E,F,C,D)
```

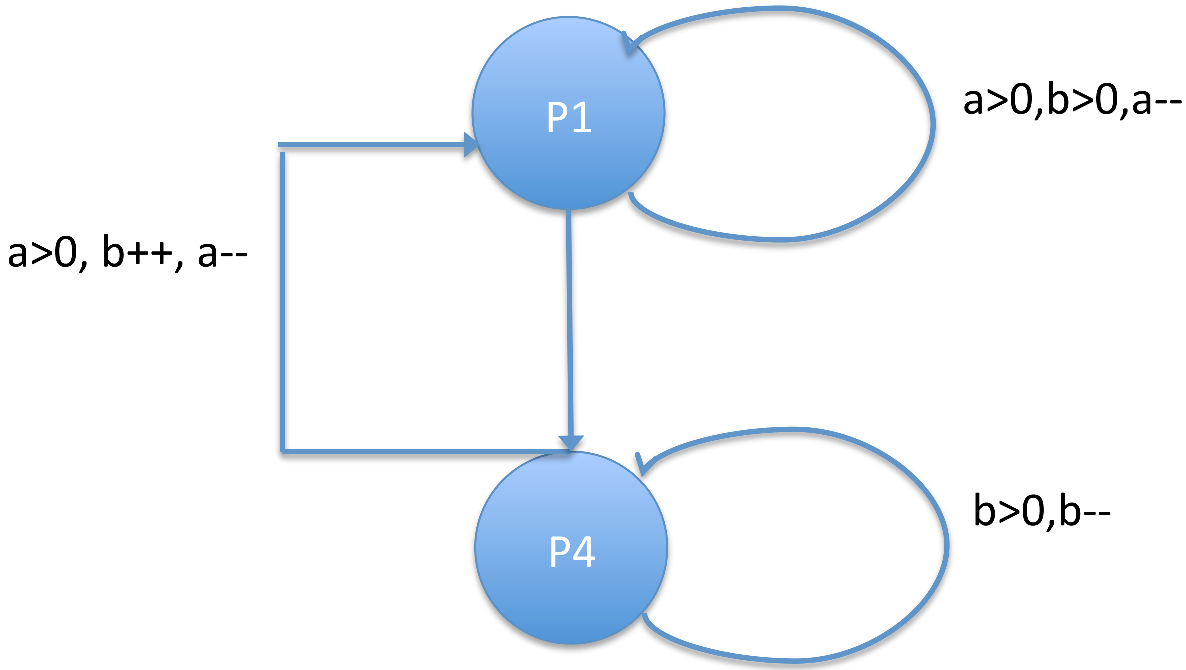
InitVal: A=c, B=d

Lex: <A,A,B>

- Bound(T3)= InitVal (B)+ Bound(T1)*increment(B,T1)+ Bound(T2)*increment(B,T2) = d+c*1+0=c+d

Overall bound = max. bound of each transition= c+d

Resource consumption



$$\sum_{i \in Loops} \text{cost\_i*bound\_i}$$

Conclusions and Future work

- Resource analysis of C programs using Horn clauses

In the future:

- Inter-procedural bound analysis
- Integration with CiaoPP for precision and non-linear bounds

Thank you!

Questions & suggestions?